SECTION A

1. What is the value of \((\cos^2 67^\circ - \sin^2 23^\circ)\)?

   Ans. \[
   \cos^2 67^\circ - \sin^2 23^\circ
   \]
   as \(\cos (90^\circ - \theta) = \sin \theta\)
   Let \(\theta = 23^\circ\)
   \[
   \cos (90^\circ - 23^\circ) = \sin 23^\circ
   \]
   \[
   \cos 67^\circ = \sin 23^\circ
   \]
   \[
   \therefore \cos^2 67^\circ = \sin^2 23^\circ
   \]
   \[
   \therefore \cos^2 67^\circ - \sin^2 23^\circ = 0
   \]

   Topic: Trigonometry_Subtopic: Complementary Angle_ Level:Easy_Std. X__CBSE Board / Mathematics

2. In an AP, if the common difference \((d) = -4\), and the seventh term \((a_7)\) is 4, then find the first term.

   Ans. \(a_7 = 4\)
   
   \[
   a + 6d = 4 \quad (\text{as } a_n = a + (n - 1)d)
   \]
   but \(d = -4\)
   \[
   a + 6(-4) = 4
   \]
   \[
   a + (-24) = 4
   \]
   \[
   a = 4 + 24 = 28
   \]
   Therefore first term \(a = 28\)

   Topic: Arithmatic Equation_Subtopic: n^{th} term_ Level: Easy_Std. X__CBSE Board / Mathematics

3. Given \(\Delta ABC \sim \Delta PQR\), if \(\frac{AB}{PQ} = \frac{1}{3}\), then find \(\frac{\text{ar } \Delta ABC}{\text{ar } \Delta PQR}\).

   Ans. \[
   \frac{A(\Delta ABC)}{A(\Delta PQR)} = \frac{AB^2}{PQ^2}
   \]
   (Ratio of area of similar triangle is equal to square of their proportional sides)
\[
\frac{A(\Delta ABC)}{A(\Delta PQR)} = \left(\frac{1}{3}\right)^2 = \frac{1}{9}
\]

**Topic: Triangle_Subtopic: Similarity_ Level: Easy_Std. X__CBSE Board / Mathematics**

4. What is the HCF of smallest prime number and the smallest composite number?

Ans. Smallest prime number is 2.
Smallest composite number is 4
Therefore HCF is 2.

**Topic:Real Numbers_Subtopic: H.C.F__ Level:Easy_Std. X__CBSE Board / Mathematics**

5. Find the distance of a point \(P(x, y)\) from the origin.

Ans. Using distance formula

\[
\ell(\text{OP}) = \sqrt{(x - 0)^2 + (y - 0)^2}
\]

**Topic:Co-ordinate Geometry_Subtopic: Distance Formula__ Level:Easy_Std. X__CBSE Board / Mathematics**

6. If \(x = 3\) is one root of the quadratic equation \(x^2 - 2kx - 6 = 0\), then find the value of \(k\).

Ans. \(\because\ x = 3\) is one of the root of \(x^2 - 2kx - 6 = 0\)

\[
(3)^2 - 2k(3) - 6 = 0
\]
\[
9 - 6k - 6 = 0
\]
\[
3 - 6k = 0
\]
\[
3 = 6k
\]
\[
k = \frac{3}{6} = \frac{1}{2}
\]

**Topic:Polynomial_Subtopic: Roots_ Level: Easy_Std. X__CBSE Board / Mathematics**

**SECTION B**

7. Two different dice are tossed together. Find the probability :

(i) of getting a doublet

(ii) of getting a sum 10, of the numbers on the two dice.

Ans. Sample space \(S = \{(1, 1), (1, 2), \ldots, (6, 6)\}\)

\[
n(s) = 36
\]

i) \(A = \text{getting a doublet}
\(A = \{(1, 1), (2, 2), \ldots, (6, 6)\}\)

\[
n(A) = 6
\]
8. Find the ratio in which \( P(4, m) \) divides the line segment joining the points \( A(2, 3) \) and \( B(6, -3) \). Hence find \( m \).

Ans. Suppose the point \( P(4, m) \) divides the line segment joining the points \( A(2, 3) \) and \( B(6, -3) \) in the ratio \( K : 1 \).

\[
\begin{align*}
A & \quad K \quad P \quad I \quad B \\
(2, 3) & \quad (4, m) & & (6, -3)
\end{align*}
\]

Co-ordinates of point \( P = \left( \frac{6K + 2}{K + 1}, \frac{-3K + 3}{K + 1} \right) \)

But the co-ordinates of point \( P \) are given as \( (4, m) \)

\[
\begin{align*}
\frac{6K + 2}{K + 1} & = 4 \quad \text{.....(1)} \\
\frac{-3K + 3}{K + 1} & = m \quad \text{.....(2)} \\
6K + 2 & = 4K + 4 \\
2K & = 2 \\
K & = 1 \\
\end{align*}
\]

Putting \( K = 1 \) in eq. (2)

\[
\frac{-3(1) + 3}{1 + 1} = m
\]

\( m = 0 \)

Ratio is 1 : 1 and \( m = 0 \)

i.e. \( P \) is the mid point of \( AB \)

9. An integer is chosen at random between 1 and 100. Find the probability that it is:

(i) divisible by 8
(ii) not divisible by 8

Ans. An integer is chosen at random from 1 to 100

Therefore \( n(S) = 100 \)

(i) Let \( A \) be the event that number chosen is divisible by 8

\[ A = \{8, 16, 24, 32, 40, 48, 56, 64, 72, 80, 88, 96\} \]

\[
\therefore \quad n(A) = 12
\]

\[
P(A) = \frac{n(A)}{n(S)} = \frac{12}{100} = \frac{3}{25}
\]

(ii) \( n(A) = 100 - 12 = 88 \)

\[
P(A') = \frac{n(A')}{n(S)} = \frac{88}{100} = \frac{22}{25}
\]
\[
\therefore \quad n(A) = 12
\]

Now, \( P(\text{that number is divisible by 8}) = P(A) = \frac{n(A)}{n(S)} \)

\[
= \frac{12}{100} = \frac{6}{50} = \frac{3}{25}
\]

\[
P(A) = \frac{3}{25}
\]

(ii) Let ‘A’ be the event that number is not divisible by 8.

\[
\therefore \quad P(A') = 1 - P(A)
\]

\[
= 1 - \frac{3}{25} = \frac{22}{25}
\]

\[
P(A') = \frac{22}{25}
\]

**Topic:** Probability **Subtopic:** Probability **Level:** Medium **Std. X** **CBSE Board / Mathematics

10. In figure 1, ABCD is a rectangle. Find the values of \( x \) and \( y \).

![Diagram of a rectangle ABCD with sides labeled x+y and x-y, and dimensions 14cm and 30cm](image)

**Ans.** Since it is a rectangle

\[
\ell(AB) = \ell(CD)
\]

\[
x + y = 30 \quad \text{...(i)}
\]

\[
\ell(AD) = \ell(BC)
\]

\[
x - y = 14 \quad \text{...(ii)}
\]

Adding (1) and (2), we get

\[
2x = 44
\]

\[
x = 22
\]

Putting \( x = 22 \) in equation (i)

\[
22 - y = 14 \Rightarrow 22 - 14 = y
\]

\[
\therefore \quad y = 8
\]

\[
\therefore \quad x = 22 \text{ and } y = 8
\]

**Topic:** Linear Equations **Subtopic:** Linear Equations **Level:** Easy **Std. X** **CBSE Board / Mathematics
11. Find the sum of first 8 multiples of 3.

Ans.

First 8 multiples of 3 are

3, 6, 9, 12, 15, 18, 21, 24

The above sequence is an A.P.

\( a = 3, \ d = 3 \) and last term \( l = 24 \)

\[ S_n = \frac{n}{2}(a + l) = \frac{8}{2}[3 + 24] = 4(27) \]

\[ S_n = 108 \]

**Topic:** Arithmetic Progression  
**Subtopic:** Sum of n term  
**Level:** Medium  
**Std. X**  
**CBSE Board / Mathematics**

12. Given that \( \sqrt{2} \) is irrational, prove that \( 5 + 3\sqrt{2} \) is an irrational number.

Ans.

Let us assume that \( 5 + 3\sqrt{2} \) is rational. Then there exist co-prime positive integers \( a \) and \( b \) such that

\[ 5 + 3\sqrt{2} = \frac{a}{b} \]

\[ 3\sqrt{2} = \frac{a}{b} - 5 \]

\[ \sqrt{2} = \frac{a - 5b}{3b} \]

\[ \Rightarrow \sqrt{2} \] is rational. [\( \because \) \( a, b \) are integers, \( \therefore \frac{a - 5b}{3b} \) is rational].

This contradicts the fact that \( \sqrt{2} \) is irrational.
So our assumption is incorrect.

Hence, \( 5 + 3\sqrt{2} \) is an irrational number.

**Topic:** Real Numbers  
**Subtopic:** Real Number  
**Level:** Difficult  
**Std. X**  
**CBSE Board / Mathematics**

**SECTION C**

13. If \( A(-2, 1), B(a, 0), C(4, b) \) and \( D(1, 2) \) are the vertices of a parallelogram \( ABCD \), find the values of \( a \) and \( b \).

Hence find the lengths of its sides.

Ans.

\( M \) is midpoint of \( AC \) and \( BD \) using midpoint formula.

\[ \left( \frac{-2 + 4}{2}, \frac{1 + b}{2} \right) = \left( \frac{a + 1}{2}, \frac{2 + 0}{2} \right) \]

\[ \left( \frac{2}{2}, \frac{1 + b}{2} \right) = \left( \frac{a + 1}{2}, \frac{2}{2} \right) \]
\[
\therefore \frac{2}{2} = \frac{a+1}{2} \Rightarrow a+1 = 2 \Rightarrow a = 1
\]
\[
\text{and} \quad \frac{1+b}{2} = \frac{2}{2} \Rightarrow 1+b = 2 \Rightarrow b = 1
\]

**Topic:** Co-ordinate Geometry  
**Subtopic:** Geometry  
**Level:** Medium  
**Std. X**  
**CBSE Board / Mathematics**

**OR**

If \(A(-5, 7), B(-4, -5), C(-1, -6)\) and \(D(4, 5)\) are the vertices of quadrilateral, find the area of the quadrilateral \(ABCD\).

Ans. \[
A(\triangle ABC) = \frac{1}{2} \left( x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2) \right)
\]

If \(A = (x_1, y_1), B = (x_2, y_2), C = (x_3, y_3)\) are vertices of \(\triangle ABC\).

\[
A(\Box ABCD) = A(\triangle ABD) + A(\triangle ADC) \quad \ldots (i)
\]

\[
A(\Box ABCD) = \frac{1}{2} \left[ -5(-5 + 6) - 4(-6 - 7) - 1(7 + 5) \right]
\]
\[
= \frac{1}{2} [-5 + 52 - 12]
\]
\[
= \frac{1}{2} [35]
\]
\[
= \frac{35}{2} \text{ Sq.units}
\]

\[
A(\triangle ADC) = \frac{1}{2} \left[ -5(5 + 6) + 4(-6 - 7) - 1(7 - 5) \right]
\]
\[
= \frac{1}{2} [-55 - 52 - 2]
\]
\[
= \frac{-109}{2}
\]

\(\therefore\) Area cannot be negative.

\[
\therefore \ A(\triangle ADC) = \frac{109}{2} \text{ sq.units}
\]
14. Find all zeroes of the polynomial \( (2x^4 - 9x^3 + 5x^2 + 3x - 1) \) if two of its zeroes are \( 2 + \sqrt{3} \) and \( 2 - \sqrt{3} \).

Ans. It is given that \( 2 + \sqrt{3} \) and \( 2 - \sqrt{3} \) are two zeros of \( f(x) = 2x^4 - 9x^3 + 5x^2 + 3x - 1 \)

\[
\left\{x-(2+\sqrt{3})\right\}\left\{x-(2-\sqrt{3})\right\} = (x-2-\sqrt{3})(x-2+\sqrt{3})
\]
\[
= (x-2)^2 - (\sqrt{3})^2
\]
\[
= x^2 - 4x + 1
\]

\( \therefore \) \( x^2 - 4x + 1 \) is a factor of \( f(x) \)

\[
\begin{array}{c|cc|cc|c}
 & 2x^4 & -9x^3 & +5x^2 & +3x & -1 \\
\hline
2x^2-x-1 & & & & & \\
\hline
& 2x^4 & -8x^3 & +2x^2 & & \\
& (-) & (+) & (-) & & \\
\hline
& -x^3 & +3x^2 & +3x & -1 & \\
& -x^3 & +4x^2 & -x & & \\
& (+) & (-) & (+) & & \\
\hline
& -x^2 & +4x & -1 & & \\
& -x^2 & +4x & -1 & & \\
& (+) & (-) & (+) & & \\
\hline
& & & & 0 & \\
\end{array}
\]

Let us now divide \( f(x) \) by \( x^2 - 4x + 1 \)

We have,

\[
\therefore \quad f(x) = (x^2 - 4x + 1)(2x^2 - x - 1)
\]

Hence, other two zeros of \( f(x) \) are the zeros of the polynomial \( 2x^2 - x - 1 \)

We have,

\[
2x^2 - x - 1 = 2x^2 - 2x + x - 1 = 2x(x - 1) + 1(x - 1)
\]
\[(2x + 1)(x - 1)\]

\[f(x) = (x - 2 - \sqrt{3})(x - 2 + \sqrt{3})(2x + 1)(x - 1)\]

Hence, the other two zeros are \(-\frac{1}{2}\) and 1.

**Topic:** Polynomial  
**Subtopic:** Roots of equation  
**Level:** Medium  
**Std. X__CBSE Board / Mathematics

15. Find HCF and LCM of 404 and 96 and verify that \(HCF \times LCM = \text{Product of the two given numbers.}\)

**Ans.** Using the factor tree for the prime factorization of 404 and 96, we have

\[404 = 2^2 \times 101\]  
\[96 = 2^5 \times 3\]

To find the HCF, we list common prime factors and their smallest exponent in 404 and 96 as under:

Common prime factor = 2, Least exponent = 2

\[\therefore \text{HCF } 2^2 = 4\]

To find the LCM, we list all prime factors of 404 and 96 and their greatest exponent as follows:

<table>
<thead>
<tr>
<th>Prime factors of 404 and 96</th>
<th>Greatest Exponent</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>5</td>
</tr>
<tr>
<td>3</td>
<td>1</td>
</tr>
<tr>
<td>101</td>
<td>1</td>
</tr>
</tbody>
</table>

\[\therefore \text{LCM } 2^5 \times 3 \times 101 = 2^5 \times 3 \times 101 = 9696\]

Now,

\[HCF \times LCM = 9696 \times 4 = 38784\]

Product of two numbers = 404 \times 96 = 38784

Therefore \(HCF \times LCM = \text{Product of two numbers.}\)

**Topic:** Real Numbers  
**Subtopic:** HCF & LCM  
**Level:** Medium  
**Std. X__CBSE Board / Mathematics

16. Prove that the lengths of tangents drawn from an external point to a circle are equal.

**Ans.** Given \(AP\) and \(AQ\) are two tangents from a point \(A\) to a circle \(C (O, r)\)

To prove \(AP = AQ\)

Construction join \(OP, OQ\) and \(OA\)

Proof In order to prove that \(AP = AQ\), we shall first prove that \(\triangle OPA = \triangle OQA\)

since a tangent at any point of a circle is perpendicular to the radius through the point of contact.

\[\because OP \perp AP \text{ and } OQ \perp AQ\]

\[\Rightarrow \angle OPA = \angle OQA = 90^\circ \text{ .......(i)}\]

Now, in right triangles \(OPA\) and \(OQA\), we have

\[\angle OPA = \angle OQA = 90^\circ \text{ .......(i)}\]

\[\therefore \triangle OPA = \triangle OQA\]

\[\therefore AP = AQ\]
17. Prove that the area of an equilateral triangle described on one side of the square is equal to half the area of the equilateral triangle described on one of its diagonal.

Ans. Let \(a\) be the side of square.

\[
A(\triangle ABC) = \frac{\sqrt{3}}{4} \times side^2 - \frac{\sqrt{3}}{4} \times a^2 \quad \ldots (1)
\]

using pythagoras theorem

\[
AD^2 = AB^2 + BD^2 = a^2 + a^2 = 2a^2
\]

\[
AD = \sqrt{2}a
\]

\[
\therefore A(\triangle ADE) = \frac{\sqrt{3}}{4} \times (\sqrt{2}a)^2 = \frac{\sqrt{3}}{4} \times 2a^2 \quad \ldots (2)
\]

\[
\frac{A(\triangle ABC)}{A(\triangle ADE)} = \frac{\sqrt{3}/4 \times a^2}{\sqrt{3}/4 \times 2a^2}
\]

\[
A(\triangle ABC) = \frac{1}{2} A(\triangle ADE)
\]

Area of equivalent triangle describes on

\[
= \frac{1}{2} \text{(area of equilateral \(\triangle\) described on one of its diagonal)}
\]

OR

If the area of two similar triangles are equal, prove that they are congruent.

Ans.
Let \( \triangle ABC \) is \( \triangle PQR \)

\[
\frac{A(\triangle ABC)}{A(\triangle PQR)} = \frac{AB^2}{PQ^2} = \frac{BC^2}{QR^2} = \frac{AC^2}{PR^2}
\]

Given that \( \triangle ABC = \triangle PQR \)

\[
\frac{A(\triangle ABC)}{A(\triangle PQR)} = 1
\]

\[
1 = \frac{AB^2}{PQ^2} = \frac{BC^2}{QR^2} = \frac{AC^2}{PR^2}
\]

\[
\therefore AB = PQ \\
BC = QR \\
AC = PR
\]

Hence corresponding sides are equal.

\[
\therefore \triangle ABC \cong \triangle PQR \quad \text{(SSS rule)}
\]

hence proved.

**Topic:** Triangle, **Subtopic:** Area, **Level:** Medium, **Std. X__CBSE Board / Mathematics

18. A plane left 30 minutes late than its scheduled time and in order to reach the destination 1500 km away in time, it had to increase its speed by 100 km/h from the usual speed. Find its usual speed.

**Ans.** Let the usual speed of the plane be \( x \) km/hr

Time taken to cover 1500 km with usual speed = \( \frac{1500}{x} \) hrs

Time taken to cover 1500 km with speed of \((x + 100)\) km/hr = \( \frac{1500}{x + 100} \) hrs.

\[
\therefore \quad \frac{1500}{x} = \frac{1500}{x + 100} + \frac{1}{2}
\]

\[
1500 \left(\frac{x + 100}{x(x + 100)}\right) = \frac{1}{2}
\]

\[
150000 \times 2 = x(x + 100)
\]
The table below shown the salaries of 280 persons:

<table>
<thead>
<tr>
<th>Salary (In thousand)</th>
<th>No. of Person</th>
</tr>
</thead>
<tbody>
<tr>
<td>5 – 10</td>
<td>49</td>
</tr>
<tr>
<td>10 – 15</td>
<td>133</td>
</tr>
<tr>
<td>15 – 20</td>
<td>63</td>
</tr>
<tr>
<td>20 – 25</td>
<td>15</td>
</tr>
<tr>
<td>25 – 30</td>
<td>6</td>
</tr>
<tr>
<td>30 – 35</td>
<td>7</td>
</tr>
<tr>
<td>35 – 40</td>
<td>4</td>
</tr>
<tr>
<td>40 – 45</td>
<td>2</td>
</tr>
<tr>
<td>45 – 50</td>
<td>1</td>
</tr>
</tbody>
</table>

Calculate the median salary of the data.

**Ans.**

<table>
<thead>
<tr>
<th>Class</th>
<th>Frequency</th>
<th>Cumulative Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>5 – 10</td>
<td>49</td>
<td>49</td>
</tr>
<tr>
<td>10 – 15</td>
<td>133</td>
<td>182</td>
</tr>
<tr>
<td>15 – 20</td>
<td>63</td>
<td>245</td>
</tr>
<tr>
<td>20 – 25</td>
<td>15</td>
<td>260</td>
</tr>
<tr>
<td>25 – 30</td>
<td>6</td>
<td>266</td>
</tr>
<tr>
<td>30 – 35</td>
<td>7</td>
<td>273</td>
</tr>
<tr>
<td>35 – 40</td>
<td>4</td>
<td>277</td>
</tr>
<tr>
<td>40 – 45</td>
<td>2</td>
<td>279</td>
</tr>
<tr>
<td>45 – 50</td>
<td>1</td>
<td>280</td>
</tr>
</tbody>
</table>

Let $N =$ total frequency

$\therefore$ we have $N = 280$

$\therefore \quad \frac{N}{2} = \frac{280}{2} = 140$

The cumulative frequency just greater than $\frac{N}{2}$ is 182 and the corresponding class is 10 – 15

$x^2 + 100x - 300000 = 0$

$x^2 + 100x - 300000 = 0$

$x = -600 \text{ or } x = 500$

But speed can’t be negative

Hence usual speed 500 km/hr.
Thus, $10 - 15$ is the median class such that

$l = 10, f = 133, F = 49$ and $h = 5$

$\text{Median} = l + \frac{N - F}{f} \times h = 10 + \left(\frac{140 - 49}{133}\right) \times 5$

$= 13.42$

**Topic:** Statistics _Subtopic: Median _ Level: Difficult _ Std. X ___ CBSE Board / Mathematics

20. A wooden article was made by scooping out a hemisphere from each end of a solid cylinder, as shown in Fig. 2. If the height of the cylinder is 10 cm and its base is of radius 3.5 cm. Find the total surface area of the article.

Ans. Let $r$ be the radius of the base of the cylinder and $h$ be its height. Then, total surface area of the article

$= \text{Curved surface area of the cylinder} + 2 \times \text{surface area of a hemisphere}$

$= 2\pi rh + 2\left(2\pi r^2\right)$

$= 2\pi r(h + 2r)$

$= 2\times \frac{22}{7} \times 3.5 \left(10 + 2 \times 3.5\right) \text{cm}^2$

$= 22 \times 17 \text{cm}^2 = 374 \text{cm}^2$

**Topic:** Area & Volume _Subtopic: Surface Area _ Level: Difficult _ Std. X ___ CBSE Board / Mathematics

OR

A heap of rice is in the form of a cone of base diameter 24 m and height 3.5 m. Find the volume of the rice. How much canvas cloth is required to just cover the heap?

Ans. Given

Base diameter = 24 m

Base radius = 12 m

Height = 3.5 m

Volume $= \frac{1}{3}\pi r^2h$
\[
\ell^2 = 12^2 + 3.5^2 = 144 + 12.25 = 156.25
\]
\[
\ell = \sqrt{156.25} = 12.5 \text{ m}
\]

Curved surface area = \(\pi r\ell\)

\[
= \frac{22}{7} \times 12 \times 12.5 = \frac{150 \times 22}{7} = 471.428 \text{ sq. meter}
\]

21. Find the area of the shaded region in Fig. 3, where arcs drawn with centres A, B, C and D intersect in pairs at mid-points P, Q, R and S of the sides AB, BC, CD and DA respectively of a square of side 12 cm, [Use \(\pi = 3.14\)]

\[
\text{Ans.} \quad \text{Given that ABCD is a square & P,Q,R & S are the mid points of AB,BC,CD & DA respectively}
\]
\[
& AB = 12 \text{ cm}
\]
\[
\Rightarrow AP = 6 \text{ cm} \quad \{ P \text{ bisects } AB \}
\]

area of the shaded region = Area of square ABCD – (Area of sector APEC + Area of sector PFQB + ....
Area of sector RGQC + Area of sector RHSD)

\[
= 12^2 - \left( \frac{\pi (6^2)}{4} + \frac{\pi 6^2}{4} + \frac{\pi 6^2}{4} + \frac{\pi 6^2}{4} \right)
\]
\[
= 12^2 - \pi \times 36
\]
= 144 – 113.04
= 30.96 cm²

**Topic:** Surface Area **Subtopic:** Area **Level:** Medium **Std. X** CBSE Board / Mathematics

22. If $4 \tan \theta = 3$, evaluate $\left( \frac{4 \sin \theta - \cos \theta + 1}{4 \sin \theta + \cos \theta - 1} \right)$

**Ans.** Given that,

$\tan \theta = \frac{3}{4}$

$\therefore \tan^2 \theta = \frac{9}{16}$

we know that,

$\sec^2 \theta = 1 + \tan^2 \theta$

$\therefore \sec^2 \theta = 1 + \frac{9}{16} = \frac{25}{16}$

$\therefore \cos^2 \theta = \frac{16}{25}$

$\therefore \cos \theta = \frac{4}{5}$

we know that,

$\sin^2 \theta = 1 - \cos^2 \theta$

$\therefore \sin^2 \theta = 1 - \frac{16}{25} = \frac{9}{25}$

$\Rightarrow \sin \theta = \frac{3}{5}$

Now,

$\left( \frac{4 \sin \theta - \cos \theta + 1}{4 \sin \theta + \cos \theta - 1} \right) = \frac{4 \times \left( \frac{3}{5} \right) - \frac{4}{5} + 1}{4 \times \frac{3}{5} + \left( \frac{4}{5} \right) - 1}$

$= \frac{12 - 4 + 5}{12 + 4 - 5}$

$= \frac{13}{11}$

**Topic:** Trigonometry **Subtopic:** Trigonometry **Level:** Medium **Std. X** CBSE Board / Mathematics
OR

If \( \tan 2A = \cot (A - 18^\circ) \), where \( 2A \) is an angle, find the value of \( A \).

Ans. Given that,

\[
\tan 2A = \cot (A - 18^\circ)
\]

Now,

we know that,

\[
\tan \theta = \cot (90^\circ - \theta)
\]

\[
\therefore \cot (90^\circ - 2A) = \cot (A - 18^\circ)
\]

\[
\therefore 90^\circ - 2A = A - 18^\circ
\]

\[
\therefore 3A = 108^\circ
\]

\[
\therefore A = \frac{108^\circ}{3} = 36^\circ
\]

\[
\therefore A = 36^\circ
\]

**Topic:** Trigonometry  
**Subtopic:** Complementary Angles  
**Level:** Easy  
**Std. X**  
**CBSE Board / Mathematics**

**SECTION D**

23. As observed from the top of a 100 m high light house from the sea-level, the angles of depression of two ships are 30° and 45°. If one ship is exactly behind the other on the same side of the light house, find the distance between the two ships. [Use \( \sqrt{3} = 1.732 \) ]

Ans. Let ships are at distance \( x \) from each other

In \( \triangle APO \)

\[
\tan 45^\circ = \frac{100}{y} = 1 \quad \therefore y = 100 \text{ m} \quad \text{(i)}
\]

In \( \triangle POB \)

\[
\tan 30^\circ = \frac{OP}{OB} = \frac{100}{x+y} = \frac{1}{\sqrt{3}}
\]

\[
\sqrt{3} = \frac{x+y}{100}
\]

\[
x + y = 100\sqrt{3} \quad \text{(ii)}
\]

\[
x = 100\sqrt{3} - y = 100\sqrt{3} - 100 = 100(\sqrt{3} - 1)
\]
16.

\[
\therefore x = 100(1.732 - 1) \\
= 100 \times 0.732 \\
= 73.2 \text{ m}
\]

\[
\therefore \text{Ships are 73.2 meters apart.}
\]

**Topic:** Trigonometry _Subtopic:_ Heights & Distance  _Level:_ Medium  _Std. X_  _CBSE Board / Mathematics

24. The diameters of the lower and upper ends of a bucket in the form of a frustum of a cone are 10 cm and 30 cm respectively. If its height is 24 cm, find:

(i) The area of the metal sheet used to make the bucket.

(ii) Why we should avoid the bucket made by ordinary plastic? [Use \( \pi = 3.14 \)]

**Ans.** Let \( r_1 = 5 \text{ cm} \) and \( r_2 = 15 \text{ cm} \) are radii of lower and upper circular faces.

Metal sheet required = Area of curved surface + Area of Base

\[
= \pi \left( r_1 + r_2 \right) \ell + \pi r_1^2
\]

... (i)

From diagram

\( AB = CD = 5 \text{ cm} \)

\( DE = 15 - 5 = 10 \text{ cm} \)

and \( BD = 24 \text{ cm} \)

\[
\therefore BE^2 = BD^2 + DE^2 \\
= 576 + 100 \\
BE^2 = 676 \\
BE = 26 \text{ cm} = \ell
\]

Metal required = \( \pi \left( 5 + 15 \right) 26 + \pi \left( 5 \right)^2 \)

\[
= \pi \times 20 \times 26 + \pi \times 25 \\
= 5\pi \left( 4 \times 26 + 5 \right) \\
= 5\pi \left( 109 \right) \\
= 5 \times \frac{22}{7} \times 109 \\
= 1712.85 \text{ cm}^2
\]

There is a chance of breakdown due to stress an ordinary plastic.

**Topic:** Area & Volume  _Subtopic:_ Surface Area  _Level:_ Difficult  _Std. X_  _CBSE Board / Mathematics
25. Prove that \( \frac{\sin A - 2\sin^3 A}{2\cos^3 A - \cos A} = \tan A \).

Ans. To prove

\[
\frac{\sin A - 2\sin^3 A}{2\cos^3 A - \cos A} = \tan A
\]

\[
L.H.S = \frac{\sin A(1 - 2\sin^2 A)}{\cos A(2\cos^2 A - 1)}
\]

We know that, \( \sin^2 A + \cos^2 A = 1 \)

\[
= \frac{\sin A\left(\sin^2 A + \cos^2 A - 2\sin^2 A\right)}{\cos A\left(2\cos^2 A - \sin^2 A - \cos^2 A\right)}
\]

\[
= \frac{\cos^2 A - \sin^2 A}{\cos^2 A - \sin^2 A}
\]

\[
= \tan A
\]

\( = R.H.S. \) hence proved.

Topic: Trigonometry _ Subtopic: Trigonometric Identities _ Level: Easy _ Std. X _ CBSE Board / Mathematics

26. The mean of the following distribution is 18. Find the frequency \( f \) of the class 19-21.

<table>
<thead>
<tr>
<th>Class</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>11-13</td>
<td>3</td>
</tr>
<tr>
<td>13-15</td>
<td>6</td>
</tr>
<tr>
<td>15-17</td>
<td>9</td>
</tr>
<tr>
<td>17-19</td>
<td>13</td>
</tr>
<tr>
<td>19-21</td>
<td>( f )</td>
</tr>
<tr>
<td>21-23</td>
<td>5</td>
</tr>
<tr>
<td>23-25</td>
<td>4</td>
</tr>
</tbody>
</table>

Ans.

\[
\sum f_i = 40 + f
\]

\[
\sum f_i u_i = f - 8
\]

we have
\( h = 2; A = 18, N = 40 + f. \sum f u_i = f - 8 \quad \bar{X} = 18 \)

\[ \therefore \text{Mean} = A + h \left( \frac{1}{N} \sum f u_i \right) \]

\[ 18 = 18 + 2 \left( \frac{1}{40 + f} \right) (f - 8) \]

\[ \frac{2(f - 8)}{40 + f} = 0 \]

\[ f - 8 = 0 \]

\[ f = 8 \]

**OR**

The following distribution gives the daily income of 50 workers of a factory:

<table>
<thead>
<tr>
<th>Daily Income (IN)</th>
<th>100-120</th>
<th>120-140</th>
<th>140-160</th>
<th>160-180</th>
<th>180-200</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of workers</td>
<td>12</td>
<td>14</td>
<td>8</td>
<td>6</td>
<td>10</td>
</tr>
</tbody>
</table>

Convert the distribution above to a less than type cumulative frequency distribution and draw its ogive.

<table>
<thead>
<tr>
<th>Daily income</th>
<th>Frequency</th>
<th>Income less than</th>
<th>Cumulative frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>100-120</td>
<td>12</td>
<td>120</td>
<td>12</td>
</tr>
<tr>
<td>120-140</td>
<td>14</td>
<td>140</td>
<td>26</td>
</tr>
<tr>
<td>140-160</td>
<td>8</td>
<td>160</td>
<td>34</td>
</tr>
<tr>
<td>160-180</td>
<td>6</td>
<td>180</td>
<td>40</td>
</tr>
<tr>
<td>180-200</td>
<td>10</td>
<td>200</td>
<td>50</td>
</tr>
</tbody>
</table>

Other than the given class intervals, we assume a class interval 80-100 with zero frequency.
27. A motor boat whose speed is 18 km/hr in still water 1 hr more to go 24 km upstream than to return downstream to the same spot. Find the speed of the stream.

Ans. Let the speed of stream be \( x \) km/hr

Now, for upstream: speed \( = (18 - x) \) km/hr

\[ \text{time taken} = \frac{24}{18 - x} \text{hr} \]

Now, for downstream: speed \( = (18 + x) \) km/hr

\[ \text{time taken} = \frac{24}{18 + x} \text{hr} \]

Given that,

\[
\frac{24}{18 - x} = \frac{24}{18 + x} + 1
\]

\[ -1 = \frac{24}{18 + x} - \frac{24}{18 - x} \]

\[ -1 = \frac{24[(18 - x) - (18 + x)]}{(18)^2 - x^2} \]

\[ -1 = \frac{24[-2x]}{324 - x^2} \]

\[ -324 + x^2 = -48x \]

\[ x^2 + 48x - 324 = 0 \]

\[ x^2 + 54x - 6x - 324 = 0 \]

\[ (x + 54)(x - 6) = 0 \]

\[ x = -54 \quad \text{or} \quad x = 6 \]

\[ x = -54 \text{ km/hr} \] (not possible)

Therefore, speed of the stream = 6 km/hr.

OR

A train travels at a certain average speed for a distance of 63 km and then travels at a distance of 72 km at an average speed of 6 km/hr more than its original speed. It takes 3 hours to complete total journey, what is the original average speed?

Ans. Let \( x \) be the original average speed of the train for 63 km.

Then, \( (x + 6) \) will be the new average speed for remaining 72 km.
Total time taken to complete the journey is 3 hrs.

\[ \frac{63}{x} + \frac{72}{x+6} = 3 \]

\[ \text{Time} = \frac{\text{Distance}}{\text{Speed}} \]

\[ \frac{63x + 378 + 72x}{x(x+6)} = 3 \]

\[ 135x + 378 = 3x^2 + 18x \]

\[ x^2 - 39x - 126 = 0 \]

\[ (x - 42)(x + 3) = 0 \]

\[ x = 42 \text{ OR } x = -3 \]

Since speed cannot be negative.
Therefore \( x = 42 \text{ km/hr.} \)

**Topic:** Polynomial Subtopic: Quadratic Equations Level: Difficult Std. X CBSE Board / Mathematics

28. The sum of four consecutive numbers in an AP is 32 and the ratio of the product of the first and the last term to the product of two middle terms is 7 : 15. Find the numbers.

**Ans.** Let the numbers be \((a, -3d), (a - d), (a + d)\) and \((a + 3d)\)

\[ (a - 3d) + (a - d) + (a + d) + (a + 3d) = 32 \]

\[ 4a = 32 \]

\[ a = 8 \]

Also,

\[ \frac{(a - 3d)(a + 3d)}{(a - d)(a + d)} = \frac{7}{15} \]

\[ 15a^2 - 135d^2 = 7a^2 - 7d^2 \]

\[ 8a^2 = 128d^2 \]

\[ d^2 = \frac{8a^2}{128} = \frac{8 \times 8 \times 8}{128} \]

\[ d^2 = 4 \]

\[ d = \pm 2 \]

If \(d = 2\) numbers are : 2, 6, 10, 14

If \(d = -2\) numbers are 14, 10, 16, 2

**Topic:** Arithmetic Progression Subtopic: A.P. Level: Medium Std. X CBSE Board / Mathematics
29. Draw a triangle \(ABC\) with \(BC = 6\) cm, \(AB = 5\) cm and \(\angle ABC = 60^\circ\). Then construct a triangle whose sides are \(\frac{3}{4}\) of the corresponding sides of the \(\Delta ABC\).

Ans. STEPS OF CONSTRUCTION:

(i) Draw a line segment \(BC = 6\) cm, draw a ray \(BX\) making \(60^\circ\) with \(BC\).
(ii) Draw an arc with radius 5 cm from \(B\) so that it cuts \(BX\) at \(A\).
(iii) Now join \(AC\) to form \(\Delta ABC\).
(iv) Draw a ray by making an acute angle with \(NC\) opposite to vertex \(A\).
(v) Locate 4 points \(B_1, B_2, B_3, B_4\) on \(BC\) by such that \(BB_1 = B_1B_2 = B_2B_3 = B_3B_4\).
(vi) Join \(B_3C\) and now draw a line from \(B_3\) parallel to \(B_4C\) so that it cuts \(BC\) at \(C'\).
(vii) From \(C'\) draw a line parallel to \(AC\) and cuts \(AB\) at \(A'\).
(viii) \(\Delta A'BC'\) is the required triangle.

30. In an equilateral \(\Delta ABC\), is a point on side \(BC\) such that \(BD = \frac{1}{3} BC\). Prove that \(9(AD)^2 = 7(AB)^2\).

Ans. Let the each side of \(\Delta ABC\) be ‘a’ unit

\[
\therefore \quad BD = \frac{a}{3}
\]

to prove : \(9(AD)^2 = 7(AB)^2\)

construction : Draw \(AM \perp BC\):

\[
DM = \frac{a}{2} - \frac{a}{3} = \frac{a}{6}
\]

\[
\therefore \quad \text{In } \Delta ABM
\]

\[
AB^2 = BM^2 + AM^2 \quad \text{......(1)}
\]
and in $\triangle ADM$
\[ AD^2 = AM^2 + DM^2 \] \hspace{1cm} \ldots \ldots \hspace{1cm} (2)

In $\triangle ABM$,
\[ \sin 60^\circ = \frac{AM}{AB} \]

$\Rightarrow$ $AM = AB \sin 60^\circ$
\[ = \frac{\sqrt{3}}{2} a \]

Now, taking $9(AD)^2$
\[ 9\left(AM^2 + DM^2\right) \]
\[ 9\left(\frac{a\sqrt{3}}{2}\right)^2 + \left(\frac{a}{6}\right)^2 \]
\[ 9\left[\frac{3a^2}{4} + \frac{a^2}{36}\right] = 9 \times \frac{28a^2}{36} \]

\[ 7(AB)^2 = 7a^2 \]

or
$\therefore$ $9(AD^2) = 7(AB^2)$ Hence proved.

**OR**

Prove that, in a right triangle, the square on the hypotenuse is equal to the sum of the squares on the other two sides.

**Ans.**

Given: A right - angled triangle $ABC$ in which $\angle B = 90^\circ$

To Prove: $(\text{Hypotenuse})^2 = (\text{Base})^2 + (\text{Perpendicular})^2$

i.e., $AC^2 = AB^2 + BC^2$

Construction from $B$ draw $BD \perp AC$.

Proof: In triangle $ADB$ and $ABC$, we have
\[ \angle ADB = \angle ABC \] \hspace{1cm} [\text{Each equal to } 90^\circ]

and, $\angle A = \angle A$ \hspace{1cm} [Common]

So, by AA - similarity criterian, we have
\[ \triangle ADB \sim \triangle ABC \]

$\Rightarrow \frac{AD}{AB} = \frac{AB}{AC}$ \hspace{1cm} [\because \text{In similar triangles corresponding sides are proportional}]
\[ \Rightarrow AB^2 = AD \times AC \quad \ldots \ldots \text{(1)} \]

In triangles BDC and ABC, we have
\[ \angle CDB = \angle ABC \quad [\text{Each equal to } 90^\circ] \]
and, \[ \angle C = \angle C \quad [\text{Common}] \]
So, by AA-similarity criterion, we have
\[ \triangle BDC \sim \triangle ABC \]
\[ \Rightarrow \frac{DC}{BC} = \frac{BC}{AC} \quad [\because \text{In similar triangles corresponding sides are proportional}] \]
\[ \Rightarrow BC^2 = AC \times DC \quad \ldots \ldots \text{(2)} \]

Adding equation (1) and (2), we get
\[ AB^2 + BC^2 = AD \times AC + AC \times DC \]
\[ \Rightarrow AB^2 + BC^2 = AC(AD + DC) \]
\[ \Rightarrow AB^2 + BC^2 = AC \times AC \]
\[ \Rightarrow AB^2 + BC^2 = AC^2 \]
Hence, \[ AC^2 = AB^2 + BC^2 \]

*Topic: Triangles* _Subtopic: Theorem_ _Level: Medium_ _Std. X_ _CBSE Board / Mathematics*