XII HSC - BOARD - 2018

Date: 03.03.2018

MATHEMATICS (40) - SOLUTIONS

SECTION - I

Q.1 (A)

(i) If $A=\begin{bmatrix}2 & -3\\ 4 & 1\end{bmatrix}$, then adjoint of matrix A is

(A) $\begin{bmatrix}1 & 3\\ -4 & 2\end{bmatrix}$ (B) $\begin{bmatrix}1 & -3\\ -4 & 2\end{bmatrix}$ (C) $\begin{bmatrix}1 & 3\\ 4 & -2\end{bmatrix}$ (D) $\begin{bmatrix}-1 & -3\\ -4 & 2\end{bmatrix}$

Ans. **(A)**

$$A=\begin{bmatrix}2 & -3\\ 4 & 1\end{bmatrix}$$

$$adjA=\begin{bmatrix}1 & 3\\ -4 & 2\end{bmatrix}$$

Topic: Matrix; Sub-topic:Adjoint__L-1__XII-HSC Board Test_Mathematics

(ii) The principal solutions of $\sec x=\frac{2}{\sqrt{3}}$ are __________.

(A) $\frac{\pi}{3},\frac{11\pi}{6}$ (B) $\frac{\pi}{6},\frac{11\pi}{6}$ (C) $\frac{\pi}{4},\frac{11\pi}{4}$ (D) $\frac{\pi}{6},\frac{11\pi}{4}$

Ans. **(B)**

$$\sec x=\frac{2}{\sqrt{3}}$$

$$\cos x=\frac{\sqrt{3}}{2}=\cos\frac{\pi}{6}=\cos\left(2\pi-\pi 6\right)$$

$$\boxed{\frac{\pi}{6},\frac{11\pi}{6}}$$

Topic: Trigo. function; Sub-topic:General solution_L-1 __XII-HSC Board Test_Mathematics

(iii) The measure of acute angle between the lines whose direction ratios are 3, 2, 6 and –2, 1, 2 is ______.

(A) $\cos^{-1}\left(\frac{1}{7}\right)$ (B) $\cos^{-1}\left(\frac{8}{15}\right)$ (C) $\cos^{-1}\left(\frac{1}{3}\right)$ (D) $\cos^{-1}\left(\frac{8}{21}\right)$

Ans. **(D)**

$$\cos\theta = \left|\frac{3\times -2+2\times 1+6\times 2}{\sqrt{(3)^2+(2)^2+(6)^2}\sqrt{(-2)^2+1^2+2^2}}\right|$$

$$=\left|\frac{-6+2+12}{\sqrt{49}\sqrt{9}}\right| = \frac{8}{7\times 3} = \frac{8}{21}$$

$$\Rightarrow \theta = \cos^{-1}\left(\frac{8}{21}\right)$$

Topic: 3D Geometry; Sub-topic:Angle__L-1 __XII-HSC Board Test_Mathematics

Q.1 (B)

(i) Wirte the negations of the following statements :

(a) All students of this college live in the hostel.

(b) 6 is an even number or 36 is a perfect square.

Ans. (a) p : All students of this college live in the hostel.

Negation :

$\sim p$: Some students of this college do not live in the hostel. **[1 Mark]**

(b) p : 6 is an even number.

q : 36 is a perfect square.

Symbolic form : $p \vee q$

$\therefore \sim(p \vee q) \equiv \sim p \wedge \sim q$

Negation :

6 is not an even number and 36 is not a perfect square. **[1 Mark]**

Topic: Logic; Sub-topic:Negation__L-1 __XII-HSC Board Test_Mathematics

(ii) If a line makes angles α, β, γ with the co-ordinates axes, prove that $\cos 2\alpha + \cos\beta + \cos 2\gamma + 1 = 0$.

Ans. L.H.S : $\cos 2\alpha + \cos 2\beta + \cos 2\gamma + 1$

$= 2\cos^2\alpha - 1 + 2\cos^2\beta - 1 + 2\cos^2\gamma - 1 + 1$ **[1 Mark]**

$= 2\left(\cos^2\alpha + \cos^2\beta + \cos^2\gamma\right) - 2$

$= 2\times 1 - 2$

$= 2 - 2$

$= 0$ **[1 Mark]**

$=$ R.H.S

Topic:3D Geometry; Sub-topic: 3D__L-1 __XII-HSC Board Test_Mathematics

(iii) Find the distance of the point $(1, 2, -1)$ from the plane $x-2y+4z-10=0$.

Ans. Distance of the point $(x_1\ y_1\ z_1)$ to plane $ax+by+cz+d=0$ is

$$D=\left|\frac{ax_1+by_1+cz_1+d}{\sqrt{a^2+b^2+c^2}}\right|$$ **[1 Mark]**

$\therefore\ (x_1\ y_1\ z_1)\equiv(1,2,-1)$

$a=1, b=-2, c=4$

$$\therefore\ D=\left|\frac{1-2(2)+4(-1)-10}{\sqrt{1+4+16}}\right|=\left|\frac{-17}{\sqrt{21}}\right|=\frac{17}{\sqrt{21}}\text{ units}$$ **[1 Mark]**

Topic:Plane; Sub-topic:Distance__L-1 __XII-HSC Board Test_Mathematics

(iv) Find the vector equation of the lines which passes through the point with position vector $4\hat{i}-\hat{j}+2\hat{k}$ and is in the direction of $-2\hat{i}+\hat{j}+\hat{k}$.

Ans. Let $\bar{a}=4\hat{i}-\hat{j}+2\hat{k}$

$\bar{b}=-2\hat{i}+\hat{j}+\hat{k}$

Equation of the line passing through point $A(\bar{a})$ and having direction $\bar{b}$ is

$\bar{r}=\bar{a}+\lambda\bar{b}$ **[1 Mark]**

$\bar{r}=(4\hat{i}-\hat{j}+2\hat{k})+\lambda(-2\hat{i}+\hat{j}+\hat{k})$ **[1 Mark]**

Topic:Line; Sub-topic:Equation__L-1 __XII-HSC Board Test_Mathematics

(v) If $\bar{a}=3\hat{i}-2\hat{j}+7\hat{k}$, $\bar{b}=5\hat{i}+\hat{j}-2\hat{k}$ and $\bar{c}=\hat{i}+\hat{j}-\hat{k}$ then find $\bar{a}\cdot(\bar{b}\times\bar{c})$.

Ans. $$\bar{a}(\bar{b}\times\bar{c})=[\bar{a}\,\bar{b}\,\bar{c}]=\begin{vmatrix}a_1 & b_1 & c_1\\ a_2 & b_2 & c_2\\ a_3 & b_3 & c_3\end{vmatrix}$$ **[1 Mark]**

$$\therefore\ \bar{a}(\bar{b}\times\bar{c})=\begin{vmatrix}3 & -2 & 7\\ 5 & 1 & -2\\ 1 & 1 & -1\end{vmatrix}$$

$= 3(-1+2)+2(-5+2)+7(5-1)$

$= 3-6+28$

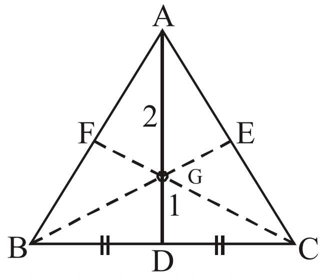
$= 25$ **[1 Mark]**

Topic: Vector; Sub-topic:Triple dot product__L-1 __XII-HSC Board Test_Mathematics

Q.2 (A)

(i) Using vector method prove that the medians of a triangle are concurrent.

Ans. Let $\bar{a}, \bar{b}, \bar{c}, \bar{d}, \bar{e}$ the position vectors of the vertices A, B, C of ΔABC and $\bar{d}, \bar{e}, \bar{f}$ be the position vectors of the midpoints D, E, F of the sides BC, CA and AB respectively



[1 Mark]

Then by the midpoint formula,

$$\bar{d} = \frac{\bar{b} + \bar{c}}{2}, \bar{e} = \frac{\bar{c} + \bar{a}}{2}, \bar{f} = \frac{\bar{a} + \bar{b}}{2}$$

$$\therefore 2\bar{d} = \bar{b} + \bar{c} \text{ ; } 2\bar{e} = \bar{c} + \bar{a} \text{ ; } 2\bar{f} = \bar{a} + \bar{b}$$

$$\therefore 2\bar{d} + \bar{a} = \bar{a} + \bar{b} + \bar{c}$$

$$2\bar{e} + \bar{b} = \bar{a} + \bar{b} + \bar{c}$$

$$2\bar{f} + \bar{c} = \bar{a} + \bar{b} + \bar{c}$$

[1 Mark]

$$\therefore \frac{2\bar{d} + \bar{a}}{2+1} = \frac{2\bar{e} + \bar{b}}{2+1} = \frac{2\bar{f} + \bar{c}}{2+1} = \frac{\bar{a} + \bar{b} + \bar{c}}{3} = \bar{g} \text{ (let)}$$

lies on the three medians AD, BE and CF dividing each of them internally in the ratio 2 : 1.
Hence, the medians are concurrent at point G . **[1 Mark]**

Topic: Vector; Sub-topic:Theorem__L-1 __XII-HSC Board Test_Mathematics

(ii) Using the truth table, prove the following logical equivalence :

$$p \leftrightarrow q \equiv (p \wedge q) \vee (\sim p \wedge \sim q)$$

Ans. **1 Mark** **1 Mark**

1	2	3	4 A	5	6	7 B	8
p	q	$p \leftrightarrow q$	$p \wedge q$	$\sim p$	$\sim q$	$\sim p \wedge \sim q$	$A \vee B$
T	T	T	T	F	F	F	T
T	F	F	F	F	T	F	F
F	T	F	F	T	F	F	F
F	F	T	F	F	T	T	T

By column number 3 and 8

$$p \leftrightarrow q \equiv (p \wedge q) \vee (\sim p \wedge \sim q)$$

[1 Mark]

Topic: Logic; Sub-topic:Truth table__L- 1__XII-HSC Board Test_Mathematics

(iii) If the origin is the centroid of the triangle whose vertices are $A(2, p, -3)$, $B(q, -2, 5)$ and $R(-5, 1, r)$, then find the values of p, q, r.

Ans. Let $\overline{a}, \overline{b}, \overline{c}$ be the position vectors of ΔABC whose vertices are $A(2, p, -3)$, $B(q, -2, 5)$, $C(-5, 1, r)$

$\therefore\ \overline{a} = 2\hat{i} + p\hat{j} - 3\hat{k},\ \overline{b} = q\hat{i} - 2\hat{j} + 5\hat{k},\ \overline{c} = -5\hat{i} + \hat{j} + r\hat{k}$

Given that origin O is the centroid of ΔABC

$\therefore\ \overline{O} = \dfrac{\overline{a}+\overline{b}+\overline{c}}{3}\quad \therefore\ \overline{a}+\overline{b}+\overline{c} = \overline{O}$ **[1 Mark]**

$2\hat{i} + p\hat{j} - 3\hat{k} + q\hat{j} - 2\hat{j} + 5\hat{k} - 5\hat{i} + \hat{j} + r\hat{k} = \overline{O}$

$\Rightarrow (2+q-5)\hat{i} + (p-2+1)\hat{j} + (-3+5+r)\hat{k} = 0\hat{i} + 0\hat{j} + 0\hat{k}$ **[1 Mark]**

by equality of vectors

$2+q-5 = 0 \Rightarrow q = 3$

$p-2+1 = 0 \Rightarrow p = 1$

$-3+5+r = 0 \Rightarrow r = -2$

$\therefore\ p = 1,\ q = 3$ and $r = -2$ **[1 Mark]**

Topic:Vector; Sub-topic:Section formula__L-1__XII-HSC Board Test_Mathematics

Q.2 (B)

(i) Show that a homogeneous equation of degree two in x and y, i.e., $ax^2 + 2hxy + by^2 = 0$ represents a pair of lines passing through the origin is $h^2 - ab \geq 0$.

Ans. Consider a homogenous equation of degree two in x and y

$$ax^2 + 2hxy + by^2 = 0 \quad .....(i)$$

In this equation at least one of the coefficients a ,b or h is non zero.

We consider two cases

Case I : If b = 0, then the equation of lines x = 0 and (ax + 2 hy) = 0 **[1 Mark]**

These lines passes through the origin.

Case II : $b \neq 0$,

Multiplying both the sides of equation (i) by b, we get **[1 Mark]**

$$abx^2 + 2hbxy + b^2y^2 = 0$$

$$b^2y^2 + 2hbxy = -abx^2$$

To make L.H.S a complete square, we add h^2x^2 on both the sides.

$$b^2y^2 + 2hbxy + h^2x^2 = -abx^2 + h^2x^2$$

$$(by+hx)^2 = (h^2-ab)x^2$$

$$(by+hx)^2 = \left[\left(\sqrt{h^2-ab}\right)x\right]^2$$

$$(by+hx)^2 - \left[\left(\sqrt{h^2-ab}\right)x\right]^2 = 0$$

$$\left[(by+hx) + \left(\sqrt{h^2-ab}\right)x\right]\left[(by+hx) - \left(\sqrt{h^2-ab}\right)x\right] = 0$$ **[1 Mark]**

It is the joint equation of two lines

$$\left(by+hx\right)+\left(\sqrt{h^2-ab}\right)x=0 \text{ and } \left(by+hx\right)-\left(\sqrt{h^2-ab}\right)x=0$$

i.e. $\left(h+\sqrt{h^2-ab}\right)x+by=0$ and $\left(h-\sqrt{h^2-ab}\right)x+by=0$

These lines passes through the origin. **[1 Mark]**

Topic: Pair of straight line; Sub-topic:Theorem__L-1__XII-HSC Board Test_Mathematics

(ii) In ΔABC, prove that $\tan\left(\frac{C-A}{2}\right)=\left(\frac{c-a}{c+a}\right)\cot\frac{B}{2}$.

Ans. In ΔABC, by sine Rule,

$$\frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C}=k$$

$\therefore\ a=k\sin A,\ b=k\sin B,\ c=k\sin C$ **[1 Mark]**

Consider,

$$\frac{c-a}{c+a}=\frac{k\sin C-k\sin A}{k\sin C+k\sin A}$$ **[1 Mark]**

$$=\frac{\sin C-\sin A}{\sin C+\sin A}$$

$$=\frac{2\cos\left(\frac{C+A}{2}\right)\sin\left(\frac{C-A}{2}\right)}{2\sin\left(\frac{C+A}{2}\right)\cos\left(\frac{C-A}{2}\right)}$$ **[1 Mark]**

$$=\cot\left(\frac{C+A}{2}\right)\cdot\tan\left(\frac{C-A}{2}\right)$$

$$=\tan\frac{B}{2}\tan\left(\frac{C-A}{2}\right)$$

$$\therefore\ \tan\left(\frac{C-A}{2}\right)=\left(\frac{C-a}{C+a}\right)\cot\frac{B}{2}$$ **[1 Mark]**

Hence proved.

Topic: Trigo. function; Sub-topic:Theorem L-1 __XII-HSC Board Test_Mathematics

(iii) Find the inverse of the matrix, $A=\begin{bmatrix}1 & 2 & -2\\ -1 & 3 & 0\\ 0 & -2 & 1\end{bmatrix}$ using elementary row transformations.

Ans. $|A|=\begin{vmatrix}1 & 2 & -2\\ -1 & 3 & 0\\ 0 & -2 & 1\end{vmatrix}$

$=1[3]-2[-1]-2[2]$

$=3+2-4=1\neq 0 \Rightarrow A^{-1}$ exists. **[1 Mark]**

We know,

$AA^{-1}=I$

$$\begin{bmatrix}1 & 2 & -2\\ -1 & 3 & 0\\ 0 & -2 & 1\end{bmatrix}A^{-1}=\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}$$ **[1 Mark]**

$R_2 \to R_2 + R_1$

$$\begin{bmatrix}1 & 2 & -2\\ 0 & 5 & -2\\ 0 & -2 & 1\end{bmatrix}A^{-1}=\begin{bmatrix}1 & 0 & 0\\ 1 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}$$

$R_2 \to R_2 + 2R_3$

$$\begin{bmatrix}1 & 2 & -2\\ 0 & 1 & 0\\ 0 & -2 & 1\end{bmatrix}A^{-1}=\begin{bmatrix}1 & 0 & 0\\ 1 & 1 & 2\\ 0 & 0 & 1\end{bmatrix}$$

$R_1 \to R_1 - 2R_2$ and $R_3 \to R_3 + 2R_2$

$$\begin{bmatrix}1 & 0 & -2\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}A^{-1}=\begin{bmatrix}-1 & -2 & -4\\ 1 & 1 & 2\\ 2 & 2 & 5\end{bmatrix}$$ **[1 Mark]**

$R_1 \to R_1 + 2R_3$

$$\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}A^{-1}=\begin{bmatrix}3 & 2 & 6\\ 1 & 1 & 2\\ 2 & 2 & 5\end{bmatrix}$$

$$\therefore\ A^{-1}=\begin{bmatrix}3 & 2 & 6\\ 1 & 1 & 2\\ 2 & 2 & 5\end{bmatrix}$$ **[1 Mark]**

Topic: Matrix; Sub-topic:Inverse__L-1 __XII-HSC Board Test_Mathematics

Q.3 (A)

(i) Find the joing equation of the pair of lines passing through the origin, which are perpendicular to the lines represented $5x^2+2xy-3y^2=0$.

Ans. Given homogeneous equation is

$5x^2+2xy-3y^2=0$

Which is factorisable

$5x^2+5xy-3xy-3y^2=0$

$5x(x+y)-3y(x+y)=0$

$(x+y)(5x-3y)=0$

$\therefore$ $x+y=0$ and $5x-3y=0$ are the two lines represented by the given equation.

$\Rightarrow$ Their slopes are -1 and $\frac{5}{3}$ **[1 Mark]**

Required two lines are respectively perpendicular to these lines.

$\therefore$ Slopes of required lines are 1 and $-\frac{3}{5}$ and the lines pass through origin.

$\therefore$ Their individual equations are

$y=1\cdot x$ and $y=-\frac{3}{5}x$

i.e., $x-y=0$ and $3x+5y=0$ **[1 Mark]**

$\therefore$ Their joint equation is

$(x-y)(3x+5y)=0$

$3x^2-3xy+5xy-5y^2=0$

$3x^2+2xy-5y^2=0$ **[1 Mark]**

Topic: Pair of straight line_Sub-topic:Formulation of equation__L-1__XII-HSC Board Test_Mathematics

(ii) Find the angle between the lines $\frac{x-1}{4}=\frac{y-3}{1}=\frac{z}{8}$ and $\frac{x-2}{2}=\frac{y+1}{2}=\frac{z-4}{1}$.

Ans. Let $\bar{a}$ and $\bar{b}$ be the vectors in the direction of the lines $\frac{x-1}{4}=\frac{y-3}{1}=\frac{z}{8}$ and $\frac{x-2}{2}=\frac{y+1}{2}=\frac{z-4}{1}$ respectively.

$\therefore$ $\bar{a}=4\hat{i}+\hat{j}+8\hat{k}$ and $\bar{b}=2\hat{i}+2\hat{j}+\hat{k}$

$\therefore$ $\bar{a}\cdot\bar{b}=4\times2+1\times2+8\times1=8+2+8=18$ **[1 Mark]**

and $|\bar{a}|=\sqrt{16+1+64}=\sqrt{81}=9$

$|\bar{b}|=\sqrt{4+1+4}=\sqrt{9}=3$

Let θ be the acute angle between the two given lines

$\therefore\ \cos\theta = \left|\dfrac{\bar{a}\cdot\bar{b}}{|\bar{a}|\cdot|\bar{b}|}\right| = \dfrac{18}{9\times 3} = \dfrac{2}{3}$ **[1 Mark]**

$\theta = \cos^{-1}\left(\dfrac{2}{3}\right)$ **[1 Mark]**

Topic: Line; Sub-topic:Line__L-1 __XII-HSC Board Test_Mathematics

(iii) Write converse, inverse and contrapositive of the following conditional statement :
If an angle is a right angle then its measure is 90°.

Ans. **Converse :** If the measure of an angle is 90° then it is a right angle. **[1 Mark]**

Inverse : If an angle is not a right angle then its measure is not 90°. **[1 Mark]**

Contra positive : If the measure of an angle is not 90° then it is not a right angle. **[1 Mark]**

Topic:Logic; Sub-topic:Condictional__L-1 __XII-HSC Board Test_Mathematics

Q.3 (B)

(i) Prove that : $\sin^{-1}\left(\dfrac{3}{5}\right)+\cos^{-1}\left(\dfrac{12}{13}\right)=\sin^{-1}\left(\dfrac{56}{65}\right)$

Ans. Let $\cos^{-1}\dfrac{12}{13} = x$

$\therefore\quad \cos x = \dfrac{12}{13}$

$\therefore\quad \sin x = \dfrac{5}{13}$ **[1 Mark]**

and let $\sin^{-1}\dfrac{3}{5} = y$

$\sin y = \dfrac{3}{5}$

$\therefore\quad \cos y = \dfrac{4}{5}$ **[1 Mark]**

$\therefore$ using $\sin(x+y) = \sin x\cos y + \cos x\sin y$ **[1 Mark]**

$= \dfrac{5}{13}\times\dfrac{4}{5}+\dfrac{12}{13}\times\dfrac{3}{5}$

$= \dfrac{20+36}{13\times 5}$

$= \dfrac{56}{65}$

$\therefore\quad x+y = \sin^{-1}\dfrac{56}{65}$

$\cos^{-1}\dfrac{12}{13}+\sin^{-1}\dfrac{3}{5}=\sin^{-1}\dfrac{56}{65}$ **[1 Mark]**

Hence proved.

Topic: Trigo. function; Sub-topic:ITF__L-1 __XII-HSC Board Test_Mathematics

(ii) Find the vector equation of the plane passing through the points $A(1, 0, 1)$, $B(1, -1, 1)$ and $C(4, -3, 2)$.

Ans. Let the p.v. of points $A(1, 0, 1)$, $B(1, -1, 1)$ and $C(4, -3, 2)$ be

$\bar{a} = \bar{i} + \bar{k},\ \bar{b} = \bar{i} - \bar{j} + \bar{k}$ and $\bar{c} = 4\bar{i} - 3\bar{j} + 2\bar{k}$

$\bar{b} - \bar{a} = -\bar{j},\ \bar{c} - \bar{a} = 3\bar{i} - 3\bar{j} + \bar{k}$ **[1 Mark]**

$$\left(\bar{b}-\bar{a}\right)\times\left(\bar{c}-\bar{a}\right) = \begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ 0 & -1 & 0 \\ 3 & -3 & 1 \end{vmatrix} = -\bar{i} + 3\bar{k}$$ **[1 Mark]**

Equation of plane through A, B, C in vector form is

$$\left(\bar{r}-\bar{a}\right)\cdot\left[\left(\bar{b}-\bar{a}\right)\times\left(\bar{c}-\bar{a}\right)\right] = 0$$ **[1 Mark]**

$$\left(\bar{r}-\bar{a}\right)\cdot\left(-\bar{i}+3\bar{j}\right) = 0$$

$$\bar{r}\cdot\left(-\bar{i}+3\bar{j}\right) = \left(\bar{i}+\bar{k}\right)\cdot\left(-\bar{i}+3\bar{j}\right) = -1+3 = 2$$

$$\therefore\ \bar{r}\cdot\left(-\bar{i}+3\bar{j}\right) = 2$$ **[1 Mark]**

Topic: Plane; Sub-topic:Equation of plane__L-1 __XII-HSC Board Test_Mathematics

(iii) Minimize $Z = 7x + y$ subject to

$5x + y \geq 5,\ x + y \geq 3,\ x \geq 0,\ y \geq 0$

Ans. $Z = 7x + y$

Subject to

$5x + y \geq 5$

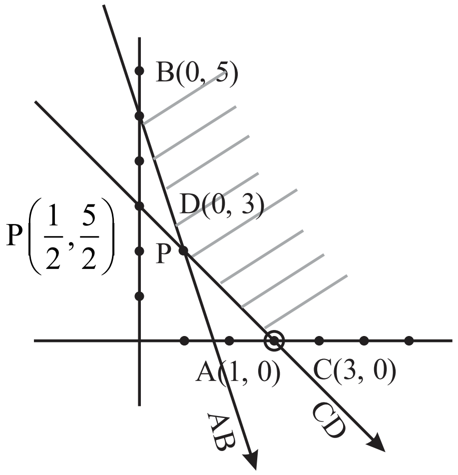
$x + y \geq 3$

$x \leq 0, y \geq 0$

Line	Inequation	Points on x	Points on y	Feasible region
AB	$5x + y \geq 5$	$A(1,0)$	$B(0,5)$	Non - origin side
CD	$x + y \geq 3$	$C(3,0)$	$D(0,3)$	Non - origin side

[1 Mark]

1 unit = 1 cm both axis



[1 Mark]

common feasible region BPC

Points	Minimize $z = 7x + y$	
$B(0,5)$	$Z(B) = 7(0)+5 = \boxed{5}$	
$P\left(\frac{1}{5},\frac{5}{2}\right)$	$Z(P) = 7\times\frac{1}{2}+\frac{5}{2} = 6$	
$C(3,0)$	$Z(C) = 7\times(3)+0 = 21$	**[1 Mark]**

Z is minimum at x = 0, y = 5 and min (z) = 5 **[1 Mark]**

Topic: LPP; Sub-topic:Graphical solution__L-1 __XII-HSC Board Test_Mathematics

SECTION - II

Q.4 (A) Select and write the appropriate answer from the given alternatives in each of the following sub-questions : [6]

(i) Let the p. m. f. of a random variable X be __

$P(x) = \frac{3-x}{10}$ for $x = -1, 0, 1, 2$

$= 0$ otherwise

Then $E(X)$ is ________.

(A) 1 (B) 2 (C) 0 (D) – 1

Ans. **(C)**

x	-1	0	1	2
$p(x)$	$\frac{4}{10}$	$\frac{3}{10}$	$\frac{2}{10}$	$\frac{1}{10}$
$x.p(x)$	$\frac{-4}{10}$	0	$\frac{2}{10}$	$\frac{2}{10}$

$\sum x.P(x) = 0$

Topic: Probability distribution; Sub-topic:Expected value__L-1 __XII-HSC Board Test_Mathematics

(ii) If $\int_0^k \frac{1}{2+8x^2}dx = \frac{\pi}{16}$, then the value of k is ________.

(A) $\frac{1}{2}$ (B) $\frac{1}{3}$ (C) $\frac{1}{4}$ (D) $\frac{1}{5}$

Ans. **(A)**

$$I = \int_0^k \frac{1}{2\left(1+(2x)^2\right)} dx = \frac{\pi}{16}$$

$$\therefore \frac{1}{2}\times\frac{1}{2}\left[\tan^{-1}(2x)\right]_0^k = \frac{\pi}{16}$$

$$\tan^{-1} 2k - \tan^{-1} 0 = \frac{\pi}{4}$$

$$2k = 1$$

$$k = \frac{1}{2}$$

Topic:Definite integral; Sub-topic:Definite integral__L-1 __XII-HSC Board Test_Mathematics

(iii) Integrating factor of linear differential equation $x\frac{dy}{dx}+2y = x^2 \log x$ is _______.

(A) $\frac{1}{x^2}$ (B) $\frac{1}{x}$ (C) x (D) x^2

Ans. **(D)**

$$\frac{dy}{dx}+\frac{2y}{x} = x\log x$$

$$P = \frac{2}{x}$$

$$I.F = e^{\int \frac{2}{x}dx} = e^{2\log x} = x^2$$

Topic: Diferential equation; Sub-topic:LDE__L-2 __XII-HSC Board Test_Mathematics

Q.4 (B) Attempt any THREE of the following : **[6]**

(i) Evaluate : $\int e^x \left[\frac{\cos x - \sin x}{\sin^2 x}\right] dx$

Ans. $I = \int e^x \left[\frac{\cos x}{\sin^2 x} - \frac{\sin x}{\sin^2 x}\right] dx$

$= \int e^x \left[\underset{f'(x)}{\cot x \cdot \operatorname{cosec} x} \quad - \underset{f(x)}{\operatorname{cosec} x}\right] dx$ **[1 Mark]**

$\because \int e^x [f(x) + f'(x)] dx = e^x f(x) + C$

$\therefore I = -e^x \cdot \operatorname{cosec} x + C$ **[1 Mark]**

Topic: Integration; Sub-topic:Integration__L-1 __XII-HSC Board Test_Mathematics

(ii) If $y = \tan^2(\log x^3)$, find $\frac{dy}{dx}$.

Ans. $y = [\tan(3\log x)]^2$

differentiate w.r.t. x both side

$\therefore \frac{dy}{dx} = 2[\tan(3\log x)] \times \sec^2(3\log x) \cdot \frac{3}{x}$ **[1 Mark]**

$\therefore \frac{dy}{dx} = \frac{6}{x}\tan(\log x^3) \cdot \sec^2(\log x^3)$ **[1 Mark]**

Topic: Differentiation; Sub-topic:Composite function__L-1 __XII-HSC Board Test_Mathematics

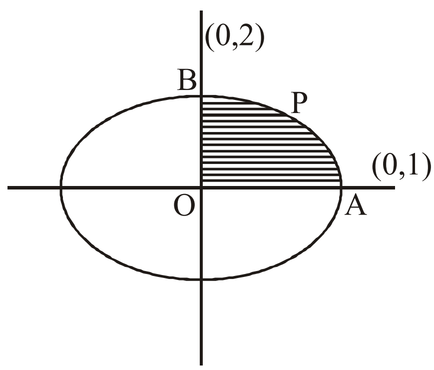
(iii) Find the area of ellipse $\frac{x^2}{1} + \frac{y^2}{4} = 1$.

Ans. Required area = 4 Area $(OAPB)$

$= \int_0^1 y\,dx$ **[1 Mark]**

$\therefore \frac{x^2}{1} + \frac{y^2}{4} = 1$

$\therefore y = 2\sqrt{1 - x^2}$



$\therefore$ Required area $= 4\int_0^1 2\sqrt{1 - x^2}\,dx$

$= 8\left[\frac{x}{2}\sqrt{1 - x^2} + \frac{1}{2}\sin^{-1}\left(\frac{x}{1}\right)\right]_0^1$

$$=8\left[\left\{0+\frac{1}{2}\sin^{-1}(1)\right\}-0\right]$$

$$=8\times\frac{1}{2}\cdot\frac{\pi}{2}=2\pi \text{ sq.units}$$ **[1 Mark]**

Topic: Definite integral; Sub-topic:AOI__L-2__XII-HSC Board Test_Mathematics

(iv) Obtain the differential equation by eliminating the arbitrary constants from the following equation:

$y=c_1 e^{2x}+c_2 e^{-2x}$

Ans. $y=c_1 e^{2x}+c_2 e^{-2x}$

differentiate w.r.t. x.

$$\frac{dy}{dx}=2c_1 e^{2x}-2c_2 e^{-2x}$$ **[1 Mark]**

Again diff. w.r.t. x.

$$\frac{d^2y}{dx^2}=4c_1e^{2x}+4c_2e^{-2x}$$

$$=4(c_1e^{2x}+c_2e^{-2x})$$

$$=4y$$

$$\therefore \frac{d^2y}{dx^2}-4y=0$$ **[1 Mark]**

Topic: Differential equation; Sub-topic:Formulation__L-1 __XII-HSC Board Test_Mathematics

(v) Given $X\sim B(n,p)$

If $n=10$ and $p=0.4$, find $E(X)$ and Var. (X).

Ans. $n=10, p=0.4 \quad q=1-p=0.6$

$E(X)=np$

$=10\times0.4=4$ **[1 Mark]**

$V(x)=npq=4\times0.6=2.4$ **[1 Mark]**

Topic:Binomial distribution; Sub-topic:Mean__L-1 __XII-HSC Board Test_Mathematics

Q.5 (A) Attempt any TWO of the following: **[6]**

(i) Evaluate: $\int \frac{1}{3+2\sin x+\cos x}dx$

Ans. Let $I=\int \frac{1}{3+2\sin x+\cos x}dx$

Put $\tan\frac{x}{2}=t$ Then, $dx=\frac{2}{1+t^2}dt,$

$\sin x=\frac{2t}{1+t^2}$ and $\cos x=\frac{1-t^2}{1+t^2}$ **[1 Mark]**

$$\therefore I=\int \frac{2dt/(1+t^2)}{3+2\left(\frac{2t}{1+t^2}\right)+\left(\frac{1-t^2}{1+t^2}\right)}$$

$$=2\int \frac{dt/(1+t^2)}{\frac{3(1+t^2)+4t+(1-t^2)}{1+t^2}}$$ **[1 Mark]**

$$=2\int \frac{dt}{2t^2+4t+4}=\int \frac{dt}{(t+1)^2+1}$$

$$=\tan^{-1}(t+1)+c$$

$$=\tan^{-1}\left[\tan\left(\frac{x}{2}\right)+1\right]+c$$ **[1 Mark]**

Topic: Integration; Sub-topic:Method of substitution__L-2 __XII-HSC Board Test_Mathematics

(ii) If $x=a\cos^3 t, y=a\sin^3 t,$

show that $\frac{dy}{dx}=-\left(\frac{y}{x}\right)^{\frac{1}{3}}$

Ans. We have, $\frac{dy}{dx}=\frac{dy/dt}{dx/dt},\quad dx/dt\neq 0$ ...(1) **[1 Mark]**

Now, $y=a\sin^3 t = a(\sin t)^3 \Rightarrow \sin t=\left(\frac{y}{a}\right)^{\frac{1}{3}}$

$$\therefore \frac{dy}{dt}=a\frac{d}{dt}(\sin t)^3=a\cdot 3(\sin t)^2\frac{d}{dt}(\sin t)$$

$=3a\sin^2 t\cos t$ ...(2)

Also, $x=a\cos^3 t = a(\cos t)^3 \Rightarrow \cos t=\left(\frac{x}{a}\right)^{\frac{1}{3}}$

$\therefore \dfrac{dx}{dt} = a \cdot 3\cos^2 t \dfrac{d}{dt}(\cos t)$

$= 3a\cos^2 t(-\sin t)$

$= -3a\cos^2 t \sin t$ ...(3) **[1 Mark]**

From (1), (2) and (3),

$\dfrac{dy}{dx} = \dfrac{3a\sin^2 t\cos t}{-3a\cos^2 t\sin t} = -\dfrac{\sin t}{\cos t} = -\left(\dfrac{y}{x}\right)^{\frac{1}{3}}$ **[1 Mark]**

Topic:Differentiation; Sub-topic:Parametric function__L-2 __XII-HSC Board Test_Mathematics

(iii) Examine the continuity of the function:

$f(x) = \dfrac{\log 100 + \log(0.01 + x)}{3x}$

$= \dfrac{100}{3}$ for $x = 0$; at $x = 0$

Ans. $\because f$ is continuous at $x = 0$ if $\lim_{x\to 0} f(x) = f(0)$. **[1 Mark]**

R.H.S. = $f(0) = \dfrac{100}{3}$ ....(given)......... ...(1) **[1 Mark]**

L.H.S. = $\lim_{x\to 0} f(x) = \lim_{x\to 0} \dfrac{\log 100 + \log(0.01 + x)}{3x} = \lim_{x\to 0} \dfrac{\log(1+100x)}{3x}$

$= \dfrac{1}{3}\lim_{x\to 0} \dfrac{[\log(1+100x)]}{100x} \times 100 = \dfrac{100}{3}$

$\Rightarrow$ From (1) and (2), L.H.S. = R.H.S.

$\therefore \lim_{x\to 0} f(x) = f(0)$.

$\therefore f$ is continuous at $x = 0$. **[1 Mark]**

Topic: Continuity; Sub-topic:At a point__L-1 __XII-HSC Board Test_Mathematics

Q.5 (B) Attempt any TWO of the following: [8]

(i) Find the maximum and minimum value of the function:

$f(x) = 2x^3 - 21x^2 + 36x - 20$

Ans. $f(x) = 2x^3 - 21x^2 + 36x - 20$

$\therefore f'(x) = 2(3x^2) - 21(2x) + 36(1) - 0$

$= 6x^2 - 42x + 36 = 6(x^2 - 7x + 6)$

$= 6(x-1)(x-6)$ **[1 Mark]**

f has a maxima/minima if $f'(x) = 0$

i.e. if $6(x-1)(x-6) = 0$

i.e. if $x - 1 = 0$ or $x - 6 = 0$

i.e. if $x = 0$ or $x = 6$ **[1 Mark]**

Now, $f''(x) = 6(2x) - 42(1) = 12x - 42$

$\therefore \quad f''(1) = 12(1) - 42 = -30$

$\therefore \quad f''(1) < 0$

Hence, f has a maximum at $x = 1$, by the second derivative test.

Also, $f''(6) = 12(6) - 42 = 30$

$\therefore \quad f''(6) > 0$

Hence, f has a minimum at $x = 6$, by the second derivative test.

Now, maximum value of f at 1,

$f(1) = 2(1^3) - 21(1^2) + 36(1) - 20$

$= 2 - 21 + 36 - 20 = -3$ **[1 Mark]**

and minimum value of f at $x = 6$,

$f(6) = 2(6^3) - 21(6^2) + 36(1) - 20$

$= 432 - 756 + 216 - 20 = -128$ **[1 Mark]**

Topic: AOD; Sub-topic:Maxima or Minima__L-1__XII-HSC Board Test_Mathematics

(ii) Prove that: $\int \frac{1}{a^2 - x^2} dx = \frac{1}{2a} \log \left| \frac{a+x}{a-x} \right| + c$

Ans. $\int \frac{1}{a^2 - x^2} dx = \int \frac{1}{(a-x)(a+x)} dx$ **[1 Mark]**

$= \frac{1}{2a} \int \frac{(a-x)+(a+x)}{(a-x)(a+x)} dx$

$= \frac{1}{2a} \int \left(\frac{1}{a+x} + \frac{1}{a-x} \right) dx$ **[1 Mark]**

$= \frac{1}{2a} \left[\int \frac{1}{a+x} dx + \int \frac{1}{a-x} dx \right]$

$= \frac{1}{2a} \left[\log|a+x| + \frac{\log|a-x|}{-1} \right] + c = \frac{1}{2a} \left[\log|a+x| - \log|a-x| \right] + c$ **[1 Mark]**

$= \frac{1}{2a} \log \left| \frac{a+x}{a-x} \right| + c$ **[1 Mark]**

Topic: Integration; Sub-topic:Theorem__L-1 __XII-HSC Board Test_Mathematics

(iii) Show that:

$$\int_{-a}^{a} f(x)dx = 2\int_{0}^{a} f(x)dx, \text{ if } f(x) \text{ is an even function}$$

$$= 0, \quad \text{if } f(x) \text{ is an odd function}$$

Ans. We shall use the following results :

$$\int_{a}^{b} f(x)\,dx = -\int_{b}^{a} f(x)\,dx \qquad ....(1)$$

$$\int_{a}^{b} f(x)dx = \int_{a}^{b} f(t)\,dt \qquad ....(2)$$

If c is between a and b, then

$$\int_{a}^{b} f(x)\,dx = \int_{a}^{c} f(x)\,dx + \int_{c}^{b} f(x)\,dx \qquad ....(3)$$

Since 0 lies between -a and a, by (3), we have,

$$\int_{-a}^{a} f(x)dx = \int_{-a}^{0} f(x)\,dx + \int_{0}^{a} f(x) = I_1 + I_2 \qquad ....(\text{Say})$$ **[1 Mark]**

In I_1, put $x = -t$. Then $dx = -dt$

When $x = -a, -t = -a \quad \therefore t = a$

When $x = 0, -t = 0 \quad \therefore t = 0$

$$\therefore \int_{-a}^{0} f(x)\,dx = \int_{a}^{0} f(-t)(-dt) = \int_{a}^{0} f(-t)\,dt$$

$$= \int_{0}^{a} f(-t)\,dt \qquad ....[By\ (1)]$$

$$= \int_{0}^{a} f(-x)\,dx \qquad ....[By\,(2)]$$

$$\therefore \int_{-a}^{a} f(x)\,dx = \int_{0}^{a} f(-x)\,dx + \int_{0}^{a} f(x)\,dx$$ **[1 Mark]**

(i) If f is an even function, then $f(-x) = f(x)$ $\therefore$ in this case,

$$\int_{-a}^{a} f(x)\,dx = \int_{0}^{a} f(x)\,dx + \int_{0}^{a} f(x)dx = 2\int_{0}^{a} f(x)\,dx$$ **[1 Mark]**

(ii) If f is an odd function, then $f(-x) = -f(x)$ $\therefore$ in this case,

$$\int_{-a}^{a} f(x)\,dx = \int_{0}^{a} -f(x)\,dx + \int_{0}^{a} f(x)\,dx = -\int_{0}^{a} f(x)\,dx + \int_{0}^{a} f(x)\,dx = 0.$$ **[1 Mark]**

Topic: Definite integral; Sub-topic:Theorem__L-1 __XII-HSC Board Test_Mathematics

Q.6 (A) Attempt any TWO of the following : **[8]**

(i) If $f(x)=\dfrac{x^2-9}{x-3}+\alpha,$ for $x>3$

$=5,$ for $x=3$

$=2x^2+3x+\beta,$ for $x<3$

is continuous at $x=3$, find α and β.

Ans. $\because$ f is continuous at $x=3$ $\quad \therefore \lim\limits_{x\to 3^-} f(x)=\lim\limits_{x\to 3^+} =f(3)$ ...(1) **[1 Mark]**

Now, $\lim\limits_{x\to 3^+} f(x) = \lim\limits_{x\to 0}\left(\dfrac{x^2-9}{x-3}+\alpha\right) = \lim\limits_{x\to 3}\left[\dfrac{(x-3)(x+3)}{(x-3)}+\alpha\right]$

$=\lim\limits_{x\to 0}[(x+3)+\alpha] = (3+3)+\alpha = \alpha+6$

and $\lim\limits_{x\to 3^-} f(x) = \lim\limits_{x\to 3}\left(2x^2+3x+\beta\right) = 2(3)^2+3(3)+\beta = 18+9+\beta = 27+\beta$

Also, $f(3)=5$ ......(given)

$\therefore$ From (1), we get, $\alpha+6=27+\beta=5$ **[1 Mark]**

$\therefore\ \alpha+6=5$ and $27+\beta=5$

$\therefore\ \alpha=5-6=-1$ and $\beta=5-27=-22$

$\Rightarrow \therefore\ \alpha=-1$ and $\beta=-22$ **[1 Mark]**

Topic: Continuity; Sub-topic:At a point__L-2__XII-HSC Board Test_Mathematics

(ii) Find $\dfrac{dy}{dx}$ if $y=\tan^{-1}\left(\dfrac{5x+1}{3-x-6x^2}\right)$

Ans. Let $y=\tan^{-1}\left(\dfrac{5x+1}{3-x-6x^2}\right)$

$=\tan^{-1}\left(\dfrac{5x+1}{1+2-x-6x^2}\right)$ **[1 Mark]**

$=\tan^{-1}\left(\dfrac{5x+1}{1-(3x+2)(2x-1)}\right)$

$=\tan^{-1}\left(\dfrac{(3x+2)+(2x-1)}{1-(3x+2)(2x-1)}\right)$

$y=\tan^{-1}(3x+2)+\tan^{-1}(2x-1)$ **[1 Mark]**

Differentiate w.r.t. x

$\therefore\ \dfrac{dy}{dx}=\dfrac{3}{1+(3x+2)^2}+\dfrac{2}{1+(2x-1)^2}$

$$=\frac{3}{1+9x^2+12x+4}+\frac{2}{1+4x^2-4x+1}$$

$$=\frac{3}{9x^2+12x+5}+\frac{1}{2x^2-2x+1}$$ **[1 Mark]**

Topic:Differentiation; Sub-topic:Inverse function__L-3__XII-HSC Board Test_Mathematics

(iii) A fair coin is tossed 9 times. Find the probability that it shows head exactly 5 times.

Ans. Let X = no. of heads shows.

$n=9 \quad p=\frac{1}{2} \quad q=\frac{1}{2}$ **[1 Mark]**

$P(X=x)={}^nC_x\, p^n.(q)^{n-x} \quad X=0,1,......,n$ **[1 Mark]**

$$P(X=5)={}^9C_5\left(\frac{1}{2}\right)^5\left(\frac{1}{2}\right)^4$$

$$=\frac{9\times8\times7\times6}{4\times3\times2\times1}\times\frac{1}{2^9}$$

$$=\frac{126}{512}$$

$=0.2460$ **[1 Mark]**

Topic:Binomial distribution; Sub-topic:Binomial distribution_L-1__XII-HSC Board Test_Mathematics

Q.6 (B) Attempt any TWO of the following : [8]

(i) Verify Rolle's theorem for the following function:

$f(x)=x^2-4x+10$ on $[0,4]$

Ans. Since $f(x)$ is a polynomial,

(i) It is continuous on [0, 4] **[1 Mark]**

(ii) It is differentiable on (0, 4) **[1 Mark]**

(iii) $f(0)=10, f(4)=16-16+10=10$

$\therefore f(0)=f(4)=10$ **[1 Mark]**

Thus all the conditions on Rolle's theorem are satisfied.

The derivative of $f(x)$ should vanish for at least one point c in (0, 4). To obtain the value of c, we proceed as follows

$$f(x) = x^2-4x+10$$

$$f'(x) = 2x-4=2(x-2)$$

$$\therefore f'(x) = 0 \Rightarrow (x-2)=0$$

$$\therefore x = 2$$

$$\therefore \exists c = 2 \text{ in } (0,4)$$

We know that $2\in(0,4)$ **[1 Mark]**

Thus Rolle's theorem is verified.

Topic:AOD; Sub-topic:Roll's theorem L-1__XII-HSC Board Test_Mathematics

(ii) Find the particular solution of the differential equation:

$$y(1+\log x)\frac{dx}{dy}-x\log x=0$$

when $y=e^2$ and $x=e.$

Ans. Given equation is

$$y(1+\log x)\frac{dx}{dy}-x\log x=0$$

$$\therefore\ y(1+\log x)\frac{dx}{dy}=x\log x$$

$$\therefore\ y(1+\log x)dx=x\log x\,dy$$

Separating the variables,

$$\frac{1}{y}dy \quad = \quad \frac{1+\log x}{x\log x}dx$$ **[1 Mark]**

Integrating, we have,

$$\int\frac{1}{y}dy \quad = \quad \int\frac{1+\log x}{x\log x}dx$$

$$\therefore \log|y| \quad = \quad \log|x\log x|+\log c$$ **[1 Mark]**

$$\therefore \log|y| \quad = \quad \log|cx\log x|$$ **[1 Mark]**

$\therefore \quad y \ = cx\log x$ is the general solution.

Given : $x \ = \ e, \quad y \ = \ e^2$

$$\therefore\ e^2 \ = c.e.\log e$$

$$\therefore\ e^2 \ = c.e.$$

$$\therefore\ c \ = e$$

$$\therefore\ y \ = ex\cdot\log x$$ **[1 Mark]**

Topic:Differential equation; Sub-topic:Variable separable method_L-1_XII-HSC Board Test_Mathematics

(iii) Find the variance and standard deviation of the random variable X whose probability distribution is given below :

x	0	1	2	3
$P(X=x)$	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{1}{8}$

Ans. **[1 Mark]**

x_i	p_i	$p_i x_i$	$p_i x_i^2$
0	$\frac{1}{8}$	0	0
1	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{3}{8}$
2	$\frac{3}{8}$	$\frac{6}{8}$	$\frac{12}{8}$
3	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{9}{8}$
	Total	$\frac{12}{8}$	$\frac{24}{8}=3$

$$E(X)=\mu=\sum p_i x_i=\frac{12}{8}=\frac{3}{2}$$ **[1 Mark]**

$$Var(X)=\sum_{i=1}^{n} p_i x_i^2-\mu^2$$

$$=3-\left(\frac{3}{2}\right)^2$$

$$=3-\frac{9}{4}$$

$$=\frac{3}{4}$$

$\therefore$ $Var(X)=\sigma^2=\frac{3}{4}$ **[1 Mark]**

Standard deivation of $(X)=\sigma_x=\sqrt{\frac{3}{4}}=\frac{\sqrt{3}}{2}$ **[1 Mark]**

Topic: Probability distribution; Sub-topic:Expected value_L-1__XII-HSC Board Test_Mathematics