SECTION - A (40 Marks)

Attempt all questions from this Section

Question 1

(a) Find the value of ‘x’ and ‘y’ if:

\[
2 \begin{bmatrix} x & 7 \\ 9 & y-5 \end{bmatrix} + \begin{bmatrix} 6 & -7 \\ 4 & 5 \end{bmatrix} = \begin{bmatrix} 10 & 7 \\ 22 & 15 \end{bmatrix}
\]

Ans.

\[
2 \begin{bmatrix} x & 7 \\ 9 & y-5 \end{bmatrix} + \begin{bmatrix} 6 & -7 \\ 4 & 5 \end{bmatrix} = \begin{bmatrix} 10 & 7 \\ 22 & 15 \end{bmatrix}
\]

\[
\begin{bmatrix} 2x & 14 \\ 18 & 2y-10 \end{bmatrix} + \begin{bmatrix} 6 & -7 \\ 4 & 5 \end{bmatrix} = \begin{bmatrix} 10 & 7 \\ 22 & 15 \end{bmatrix}
\]

\[
\begin{bmatrix} 2x+6 & 14-7 \\ 18+4 & 2y-10+5 \end{bmatrix} = \begin{bmatrix} 10 & 7 \\ 22 & 15 \end{bmatrix}
\]

Using equality of matrix

\[
2x + 6 = 10 \quad \text{and} \quad 2y - 10 + 5 = 15
\]

\[
\therefore 2x = 4 \quad \text{and} \quad 2y = 15
\]

\[
\therefore x = 2 \quad \text{and} \quad y = 20
\]

\[
y = 10
\]

(b) Sonia had a recurring deposit account in a bank and deposited Rs. 600 per month for 2\(\frac{1}{2}\) years. If the rate of interest was 10% p.a., find the maturity value of this account.

Ans.

Recurring deposit = 600 per month

Period = 2\(\frac{1}{2}\) yrs = 30 months

R.O.I = (r) = 10%

Total principal per 1 month

\[
\text{Total principal} = 600 \times 30 = 18000
\]

\[
\text{Maturity value} = \frac{18000 \times (1 + 0.10)^{2.5} - 1}{0.10}
\]

\[
\text{Maturity value} = \frac{18000 \times 1.10^{2.5} - 1}{0.10}
\]

\[
\text{Maturity value} = \frac{18000 \times 1.6289 - 1}{0.10}
\]

\[
\text{Maturity value} = \frac{29326.2}{0.10}
\]

\[
\text{Maturity value} = 293262
\]

\[
\text{Maturity value} = \text{Rs. 293,262}
\]
\[
= 600 \left( \frac{n(n+1)}{2} \right) = 300 (30 \times 31) = \text{Rs.2,79,000}
\]

\[
\therefore \text{Interest} = \frac{PRT}{100} = \frac{279000 \times 10 \times 1}{100 \times 12} = \text{Rs.2325}
\]

\[
\therefore \text{Maturity value} = 600 \times 30 + 2325 = \text{Rs.20325}
\]

**Topic: Banking**

**Subtopic: Recurring Deposits**

**Level: 1**

**Std. X**

**ICSE Board**

**/ Mathematics**

(c) Cards bearing numbers 2, 4, 6, 8, 10, 12, 14, 16, 18 and 20 are kept in a bag. A card is drawn at random from the bag. Find the probability of getting a card which is: [4]

(i) a prime number

(ii) a number divisible by 4

(iii) a number that is a multiple of 6

(iv) an odd number

Ans.

\[
\begin{array}{cccccc}
2 & 4 & 6 & 8 & 10 & 12 \\
14 & 16 & 18 & 20 & \\
\end{array}
\]

\[
n(S) = ^{10}C_1 = 10
\]

(i) \( A = \text{a prime number} = \{2\} \)

\[
P(A) = \frac{1}{10}
\]

(ii) \( B = \text{Number divisible by 4} = \{4, 8, 12, 16, 20\} \)

\[
P(B) = \frac{5}{10} = \frac{1}{2}
\]

(iii) \( C = \text{a number that is multiple of 6} \)

\[
= \{6, 12, 18\}
\]

\[
P(C) = \frac{3}{10}
\]

(iv) \( D = \text{an odd number} = \{\} \)

\[
P(D) = \frac{0}{10} = 0
\]

**Topic: Probability**

**Subtopic: Probability**

**Level: 1**

**Std. X**

**ICSE Board**

**/ Mathematics**
Question 2

(a) The circumference of the base of a cylindrical vessel is 132 cm and its height is 25 cm. Find the [3]

(i) radius of the cylinder

(ii) volume of cylinder \( \text{Use } \pi = \frac{22}{7} \)

Ans. (i) Given circumference = \( 2\pi r \)

\[
132 = 2 \times \frac{22}{7} \times r
\]

\[\therefore r = \frac{21}{2} \times 7 = 21 \text{ cm} \]

\[\therefore \text{Radius} = 21 \text{ cm} \]

(ii) Volume of cylinder = \( \pi r^2 h \)

\[
= \frac{22}{7} \times 21 \times 21 \times 25
\]

\[= 22 \times 21 \times 25 \times 3 \]

\[= 34,650 \text{ cm}^3 \]

**Topic: Mensuration **

Subtopic: Volume of cylinder

Level: 1

Std. X

ICSE Board / Mathematics

(b) If \((k – 3), (2k + 1)\) and \((4k + 3)\) are three consecutive terms of an A.P., find the value of \(k\). [3]

Ans. \( \because (k – 3), (2k + 1), (4k + 3)\) are consecutive numbers in AP.

\[\therefore 2 \times (2k + 1) = (k – 3) + (4k + 3) \]

\[\therefore 4k + 2 = k – 3 + 4k + 3 \]

\[\therefore k = 2 \]

**Topic: Progression **

Subtopic: A.P.

Level: 1

Std. X

ICSE Board / Mathematics

(c) \(PQRS\) is a cyclic quadrilateral. Given \(\angle QPS = 73^\circ, \angle PQS = 55^\circ\) and \(\angle PSR = 82^\circ\), calculate :[4]

(i) \(\angle QRS\)

(ii) \(\angle RQS\)

(iii) \(\angle PRQ\)
Ans. From diagram

(i) \( \angle SPQ + \angle QRS = 180^\circ \) (Opposite angles are supplementary)

\[
73^\circ + \angle QRS = 180^\circ \\
\angle QRS = 180^\circ - 73^\circ = 107^\circ
\]

(ii) \( \angle PSR + \angle PQR = 180^\circ \)

\[
82^\circ + \angle PQR = 180^\circ \\
\angle PQR = 180^\circ - 82^\circ \\
\angle PQR = 98^\circ
\]

But \( \angle PQR = \angle PQS + \angle RQS \)

\[
98^\circ = 55^\circ + \angle RQS \\
98^\circ - 55^\circ = \angle RQS = 43^\circ
\]

(iii)

\[
\therefore \angle PRQ = 52^\circ
\]

**Topic: Circles_Subtopic:Circles_ Level:2 _Std. X__ICSE Board / Mathematics**

**Question 3**

(a) If \((x + 2)\) and \((x + 3)\) are factors \(x^3 + ax + b\), find the values of ‘a’ and ‘b’.  

**Ans.**

\[
\therefore x + 2 \text{ is factor of } x^3 + ax + b
\]

\[
\therefore (-2)^3 + a(-2) + b = 0 \\
-8 - 2a + b = 0 \\
2a - b = -8 \quad \text{...(i)}
\]

\[
\therefore x + 3 \text{ is factor of } x^3 + ax + b
\]

\[
(-3)^3 + a(-3) + b = 0 \\
-27 - 3a + b = 0
\]
3a - b = -27 ...(ii)
(ii) - (i)

(3a - b) - (2a - b) = -27 - (-8)

3a - b - 2a + b = -27 + 8

a = -19

Put a = -19 in (i)

2(-19) - b = -8

-38 - b = -8

-38 + 8 = b = -30

(b) Prove that \( \sqrt{\sec^2 \theta + \cosec^2 \theta} = \tan \theta + \cot \theta \)

Ans.

L.H.S

\[ \sqrt{\sec^2 \theta + \cosec^2 \theta} \]

\[ \therefore \sqrt{\tan^2 \theta + 1 + \cot^2 \theta + 1} \]

\[ \therefore \sqrt{\tan^2 \theta + 2 + \cot^2 \theta} \]

\[ \therefore \sqrt{\tan^2 \theta + 2 \tan \theta \cot \theta + \cot^2 \theta} \]

\{\tan \theta \times \cot \theta = 1\} \]

\[ \therefore \sqrt{(\tan \theta + \cot \theta)^2} \]

\[ \therefore \tan \theta + \cot \theta \]

(c) Using graph paper draw a histogram for the given distribution showing the number of runs scored by 50 batsman. Estimate the mode of the data :

<table>
<thead>
<tr>
<th>Runs Scored</th>
<th>No. of batsman</th>
</tr>
</thead>
<tbody>
<tr>
<td>1000-4000</td>
<td>4</td>
</tr>
<tr>
<td>4000-5000</td>
<td>18</td>
</tr>
<tr>
<td>5000-6000</td>
<td>9</td>
</tr>
<tr>
<td>6000-7000</td>
<td>6</td>
</tr>
<tr>
<td>7000-8000</td>
<td>7</td>
</tr>
<tr>
<td>8000-9000</td>
<td>2</td>
</tr>
<tr>
<td>9000-10000</td>
<td>4</td>
</tr>
</tbody>
</table>

Ans.
Question 4

(a) Solve the following inequation, write down the solution set and represent it on the real number line:

\[-2 + 10x \leq 13x + 10 < 24 + 10x, x \in \mathbb{Z} \]

Ans.

\[-2 + 10x \leq 13x + 10 \text{ and } 13x + 10 < 24 + 10x\]
\[-12 \leq 3x \text{ and } 3x < 14\]

\[-4 \leq x \text{ and } x < \frac{14}{3} \quad \therefore x \in \mathbb{Z}\]

\[\therefore x = \{-4,-3,-2,-1,0,1,2,3,4\}\]

(b) If the straight lines \(3x - 5y = 7\) and \(4x + ay + 9 = 0\) are perpendicular to one another, find the value of \(a\).
Ans.  
Slope of $3x - 5y = 7$ is

$$m_1 = \frac{-3}{-5} = \frac{3}{5}$$

Slope of $4x + ay + 9 = 0$ is

$$m_2 = \frac{-4}{a}$$

$\therefore$ lines are $\perp$

$$m_1 \times m_2 = -1$$

$$\frac{3}{5} \times \frac{-4}{a} = -1$$

$$\frac{-12}{5} = -a$$

$$\therefore a = \frac{12}{5}$$

**Topic:** Coordinate Geometry _Subtopic:_ Equation of line _Level:_ 1 _Std:_ X _ICSE Board / Mathematics

(c) Solve $x^2 + 7x = 7$ and give your answer correct to two decimal places.

Ans.  

$x^2 + 7x = 7$

$$x^2 + 7x + \frac{49}{4} = 7 + \frac{49}{4}$$

$$\left(x + \frac{7}{2}\right)^2 = \frac{77}{4}$$

$$x + \frac{7}{2} = \pm \sqrt{\frac{77}{4}}$$

$$x = \pm \frac{\sqrt{77}}{2} - \frac{7}{2}$$

$$x = \pm \frac{\sqrt{77} - 7}{2}$$

$$x = 0.88 \text{ or } x = -7.88$$

**Topic:** Algebra _Subtopic:_ Quadratic Equation _Level:_ 1 _Std:_ X _ICSE Board / Mathematics
SECTION - B (40 Marks)

Attempt any four questions from this Section

Question 5

(a) The 4th term of a G.P. is 16 and the 7th terms is 128. Find the first term and common ratio of the series.

\[a_4 = ar^3 = 16\]  
\[a_7 = ar^6 = 128\]

\[(ii) \div (i)\]

\[\frac{ar^6}{ar^3} = \frac{128}{16}\]

\[r^3 = 8\]

\[r = 2\]

Put \(r = 2\) in equation (ii)

\[a(2)^3 = 16\]

\[a = 2\]

\[\therefore\ \text{First term (a) = 2}\]

\[\text{Common ratio} = 2\]

(b) A man invests Rs.22,500 in Rs.50 shares available at 10% discount. If the dividend paid by the company is 12%, calculate:

(i) The number of shares purchased

(ii) The annual dividend received

(iii) The rate of return he gets on his investment. Give your answer correct to the nearest whole number.

\[\text{Actual price} = 50 \text{ Rs./Share}\]

\[\text{Price after discount} = 50 - 10\% \text{ of } 50 = 45\text{ Rs.}\]

(i) Total shares bought (Purchased)

\[= \frac{22500}{45} = 500\]

(ii) Annual dividend received

\[= 500 \times 50 \times 12\%\]
= 500 \times 50 \times \frac{12}{100} \\
= 5 \times 600 \\
= \text{Rs. 3000}

(iii) \quad \text{ROR} = \frac{25500 - 22500}{22500} \times 100 \\
= \frac{3000}{22500} \times 100 \approx 13.33\%

Topic: Shares Subtopic: Shares Level: 2 Std. X ICSE Board / Mathematics

(c) Use graph paper for this question (Take 2cm = 1 unit along both x and y axis). \(ABCD\) is a quadrilateral whose vertices are \(A(2, 2), B(2, -2), C(0, -1)\) and \(D(0, 1)\).

(i) Reflect quadrilateral \(ABCD\) on the y-axis and name it as \(A'B'CD\).

(ii) Write down the coordinates of \(A'\) and \(B'\).

(iii) Name two points which are invariant under the above reflection.

(iv) Name the polygon \(A'B'CD\).

Ans.

\(A' = (-2, 2) \quad B' = (-2, -2)\)

\(C\) and \(D\)

\(A'B'CD\) is trapezium

Topic: Coordinate Geometry Subtopic: Reflection Level: 2 Std. X ICSE Board / Mathematics
Question 6

(a) Using properties of proportion, solve for \( x \). Given that \( x \) is positive:

\[
\frac{2x + \sqrt{4x^2 - 1}}{2x - \sqrt{4x^2 - 1}} = 4
\]

Ans. \( \frac{2x + \sqrt{4x^2 - 1}}{2x - \sqrt{4x^2 - 1}} = \frac{4}{1} \)

Applying componendo and dividendo

\[
\frac{(2x + \sqrt{4x^2 - 1}) + (2x - \sqrt{4x^2 - 1})}{(2x + \sqrt{4x^2 - 1}) - (2x - \sqrt{4x^2 - 1})} = \frac{4 + 1}{4 - 1}
\]

\[ \Rightarrow \frac{4x}{2\sqrt{4x^2 - 1}} = \frac{5}{3} \]

Squaring on both sides, we get

\[ \Rightarrow \frac{4x^2}{4x^2 - 1} = \frac{25}{9} \]

\[ \Rightarrow 36x^2 = 100x^2 - 25 \]

\[ \Rightarrow 64x^2 = 25 \]

\[ \Rightarrow x^2 = \frac{25}{64} \]

\[ x = \pm \frac{5}{8} \]

(b) If \( A = \begin{bmatrix} 2 & 3 \\ 5 & 7 \end{bmatrix}, \ B = \begin{bmatrix} 0 & 4 \\ -1 & 7 \end{bmatrix} \) and \( C = \begin{bmatrix} 1 & 0 \\ -1 & 4 \end{bmatrix} \), find \( AC + B^2 - 10C \). 

Ans. \( A = \begin{bmatrix} 2 & 3 \\ 5 & 7 \end{bmatrix}, \ B = \begin{bmatrix} 0 & 4 \\ -1 & 7 \end{bmatrix} \) and \( C = \begin{bmatrix} 1 & 0 \\ -1 & 4 \end{bmatrix} \)

\[
AC + B^2 - 10C = \begin{bmatrix} 2 & 3 \\ 5 & 7 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -1 & 4 \end{bmatrix} + \begin{bmatrix} 0 & 4 \\ -1 & 7 \end{bmatrix} \begin{bmatrix} 0 & 4 \\ -1 & 7 \end{bmatrix} - 10 \begin{bmatrix} 1 & 0 \\ -1 & 4 \end{bmatrix}
\]

\[
AC + B^2 - 10C = \begin{bmatrix} 2 & 3 \\ 5 & 7 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -1 & 4 \end{bmatrix} + \begin{bmatrix} 0 & 4 \\ -1 & 7 \end{bmatrix} \begin{bmatrix} 0 & 4 \\ -1 & 7 \end{bmatrix} - 10 \begin{bmatrix} 1 & 0 \\ -1 & 4 \end{bmatrix}
\]

\[
AC + B^2 - 10C = \begin{bmatrix} 2 & 3 \\ 5 & 7 \end{bmatrix} + \begin{bmatrix} 0 & 4 \\ -1 & 7 \end{bmatrix} - 10 \begin{bmatrix} 1 & 0 \\ -1 & 4 \end{bmatrix}
\]

\[
AC + B^2 - 10C = \begin{bmatrix} 2 & 3 \\ 5 & 7 \end{bmatrix} + \begin{bmatrix} 0 & 4 \\ -1 & 7 \end{bmatrix} - 10 \begin{bmatrix} 1 & 0 \\ -1 & 4 \end{bmatrix}
\]
\[ AC + B^2 - 10C = \begin{bmatrix} -1 - 4 - 10 & 12 + 28 \\ -2 - 7 + 10 & 28 + 45 - 40 \end{bmatrix} \]

\[ AC + B^2 - 10C = \begin{bmatrix} -15 & 40 \\ 1 & 33 \end{bmatrix} \]

**Topic:** Algebra Subtopic: Matrices _Level: 1 _Std. X__ICSE Board / Mathematics

(c) Prove that \((1 + \cot \theta - \cosec \theta)(1 + \tan \theta + \sec \theta) = 2\) \[\text{[4]}\]

**Ans.** Taking LHS:

\[(1 + \cot \theta - \cosec \theta)(1 + \tan \theta + \sec \theta)\]

\[= \left(1 + \frac{\cos \theta}{\sin \theta} - \frac{1}{\sin \theta}\right)\left(1 + \frac{\sin \theta}{\cos \theta} + \frac{1}{\cos \theta}\right)\]

\[= \frac{(\sin \theta + \cos \theta - 1)(\sin \theta + \cos \theta + 1)}{\sin \theta \cos \theta}\]

\[= \frac{(\sin \theta + \cos \theta)^2 - 1^2}{\sin \theta \cdot \cos \theta}\]

\[= \frac{\sin^2 \theta + \cos^2 \theta + 2\sin \theta \cdot \cos \theta - 1}{\sin \theta \cdot \cos \theta}\]

\[= \frac{1 + 2\sin \theta \cdot \cos \theta - 1}{\sin \theta \cdot \cos \theta}\]

\[= \frac{2\sin \theta \cdot \cos \theta}{\sin \theta \cdot \cos \theta}\]

\[= 2\]

**Topic:** Trigonometry _Subtopic: Trigonometrical Identities _Level: 2 _Std. X__ICSE Board / Mathematics

**Question 7**

(a) Find the value of \(k\) for which the following equation has equal roots. \[\text{[3]}\]

\[x^2 + 4kx + \left(k^2 - k + 2\right) = 0\]

**Sol.** \[x^2 + 4kx + \left(k^2 - k + 2\right) = 0\]

following equation having equal roots

\[\therefore \ b^2 - 4ac = 0\]

here \(a = 1, \ b = 4k, \ c = k^2 - k + 2\)

\[(4k)^2 - 4(1)(k^2 - k + 2) = 0\]
\[ 16k^2 - 4k^2 + 4k - 8 = 0 \]
\[ 12k^2 + 4k - 8 = 0 \]
\[ 3k^2 + k - 2 = 0 \]
\[ 3k^2 + 3k - 2k - 2 = 0 \]
\[ 3k(k + 1) - 2(k + 1) = 0 \]
\[ (k+1)(3k-2) = 0 \]
\[ k = -1 \quad \text{or} \quad k = \frac{2}{3} \]

**Topic:** Algebra \_ **Subtopic:** Quadratic Equation \_ **Level:** 2 \_ **Std. X** \_ **ICSE Board** \_ **Mathematics**

(b) On a map drawn to a scale of 1 : 50,000, a rectangular plot of land ABCD has the following dimensions. 
AB = 6cm, BC = 8 cm and all angles are right angles. Find:
(i) the actual length of the diagonal distance AC of the plot in km.
(ii) the actual area of the plot in sq km.

**Sol.**

\[ \text{Using pythagorean theorem} \]
\[ AC^2 = AB^2 + BC^2 \]
\[ AC^2 = 6^2 + 8^2 \]
\[ AC^2 = 100 \]
\[ AC = 10 \text{ cm} \]

1) \[ \therefore \] Actual length
\[ = 10 \times 50000 = 500000 \text{ cm} \]
\[ = \frac{500000}{1000 \times 100} = 5 \text{ km} \]

2) Area of ABCD = 6 \times 8
\[ = 48 \text{ cm}^2 \]
\[ = \frac{48 \times 50000 \times 50000}{100000 \times 100000} \]
\[ = \frac{48}{4} = 12 \text{ sq. km} \]

**Topic:** Mensuration \_ **Subtopic:** Area \_ **Level:** 2 \_ **Std. X** \_ **ICSE Board** \_ **Mathematics**

(c) A (2, 5), B(-1, 2) and C(5, 8) are the vertices of a triangle ABC, ‘M’ is a point on AB such that AM : MB = 1 : 2. Find the co-ordinates of ‘M’. Hence find the equation of the line passing through the points C and M.
Sol. A(2,5), B(-1,2) and C(5,8)

Let the co-ordinates of M is (x, y)

\[x = \frac{2 \times 2 + 1 \times (-1)}{2 + 1} = \frac{4 - 1}{3} = 1\]

\[y = \frac{2 \times 5 + 1 \times 2}{2 + 1} = \frac{12}{3} = 4\]

:. point M = (1, 4)

Equation of line passing through C (5, 8) and M (1, 4).

\[y - 8 = \frac{4 - 8}{1 - 5}(x - 5)\]

\[y - 8 = \frac{-4}{-4}(x - 5)\]

\[y - 8 = 1(x - 5)\]

\[y - 8 = x - 5\]

\[x - y + 3 = 0\]

**Topic: Coordinate Geometry_Subtopic: Equation of line__ Level:2 _Std. X__ICSE Board / Mathematics**

**Question 8**

(a) Rs. 7500 were divided equally among a certain number of children. Had there been 20 less children, each would have received Rs. 100 more. Find the original number of children. [3]

**Sol.** Let the original number of person be \(x\), then 7500 divided equally between \(x\) person,

each one get = \(\frac{7500}{x}\)

7500 divided equally between \(x\) - 20 children

each one get \(75 = \frac{7500}{x - 20}\)
According to the question

\[ \frac{7500}{x - 20} = \frac{7500 + 100}{x} \]

\[ \frac{7500}{x - 20} = \frac{7500 + 100x}{x} \]

\[ 7500x = (x - 20)(7500 + 100x) \]

\[ 75x = (x - 20)(75 + x) \]

\[ 75x = 75x + x^2 - 1500 - 20x \]

\[ x^2 - 20x - 1500 = 0 \]

\[ x = \frac{20 \pm \sqrt{400 - 4(-1500)}}{2} \]

\[ x = \frac{20 \pm \sqrt{400 + 6000}}{2} \]

\[ x = \frac{20 \pm 80}{2} \]

\[ x = \frac{20 + 80}{2} \quad \text{or} \quad x = \frac{20 - 80}{2} \]

\[ x = 50 \quad \text{or} \quad x = -30 \quad \text{(not possible)} \]

\[ \therefore \text{original number of children} = 50 \]

**Topic: Algebra**

**Subtopic: Quadratic Equation**

**Level: 1**

**Std. X**

**ICSE Board / Mathematics**

(b) If the mean of the following distribution is 24, find the value of ‘a’

<table>
<thead>
<tr>
<th>Marks</th>
<th>0 - 10</th>
<th>10 - 20</th>
<th>20 - 30</th>
<th>30 - 40</th>
<th>40 - 50</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of students</td>
<td>7</td>
<td>a</td>
<td>8</td>
<td>10</td>
<td>5</td>
</tr>
</tbody>
</table>

Sol. Mean = 24

<table>
<thead>
<tr>
<th>Class</th>
<th>Frequency (f_i)</th>
<th>Class mark (x_i)</th>
<th>f_i × x_i</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 - 10</td>
<td>7</td>
<td>5</td>
<td>35</td>
</tr>
<tr>
<td>10 - 20</td>
<td>a</td>
<td>15</td>
<td>15a</td>
</tr>
<tr>
<td>20 - 30</td>
<td>8</td>
<td>15</td>
<td>200</td>
</tr>
<tr>
<td>30 - 40</td>
<td>10</td>
<td>35</td>
<td>350</td>
</tr>
<tr>
<td>40 - 50</td>
<td>5</td>
<td>45</td>
<td>225</td>
</tr>
<tr>
<td>Total</td>
<td>30 + a</td>
<td></td>
<td>810 + 15a</td>
</tr>
</tbody>
</table>
\[ \text{Mean} = \frac{\sum f_i x_i}{\sum f_i} = 24 \]

\[ \frac{810 + 15a}{30 + a} = 24 \]

\[ 810 + 15a = 720 + 24a \]

\[ 90 = 9a \]

\[ a = 10 \]

**Topic: Statistics_Subtopic: Mean_ Level: 1_Std. X__ICSE Board / Mathematics**

(c) Using ruler and compass only, construct a \( \triangle ABC \) such that \( BC = 5 \text{ cm} \) and \( AB = 6.5 \text{ cm} \) and \( \angle ABC = 120^\circ \)

(i) Construct a circum-circle of \( \triangle ABC \)

(ii) Construct a cyclic quadrilateral ABCD, such that D is equidistant from AB and BC.

**Sol.**

Step of construction:

(i) Draw \( BC = 5 \text{ cm} \)

(ii) At B, draw \( \angle XBC = 120^\circ \)

(iii) From BX, cut off \( AB = 6.5 \text{ cm} \)

(iv) Join AC to get \( \triangle ABC \)

(v) Draw the perpendicular bisector of BC and AB. These bisectors meet at O. With O as centre and radius equal to OA, draw a circle, which passes through A, B and C. This is the required circumcircle of \( \triangle ABC \)

(vi) Produce the perpendicular bisector of BC so that it meets the circle at D. Join CD and AD to get the required cyclic quadrilateral ABCD.

**Topic: Geometry_Subtopic: Construction_ Level:1 _Std. X__ICSE Board / Mathematics**

Question 9

(a) Priyanka has a recurring deposit account of Rs. 1000 per month at 10% per annum. If she gets Rs. 5550 as interest at the time of maturity, find the total time for which the account was held.

**Sol.**

Amount of recurring deposit per month = Rs. 1000

Rate of interest = 10% p.a.

let period = \( n \) months

Amount of interest = 5550

Total principal for one month = \[ \frac{1000 \times n(n+1)}{2} \] ......(1)

Interest = \[ \frac{1000n(n+1)}{2} \times \frac{10}{100} \times \frac{1}{12} \]

\[ = \frac{25}{6} n(n+1) \] ......(2)

From (1) nd (2), we get

\[ \frac{25}{6} n(n+1) = 5550 \]
\[
25n^2 + 25n = 33300 \\
25n^2 + 25n - 33300 = 0 \\
n^2 + n - 1332 = 0 \\
n^2 + 37n - 36n - 1332 = 0 \\
n(n + 37) - 36(n + 37) = 0 \\
(n - 36)(n + 37) = 0 \\
\]
\[n = 36\]

**Topic: Commercial Arithmetic_Subtopic: Banking__ Level:2__ Std. X__ ICSE Board / Mathematics**

(b) In \(\triangle PQR\), \(MN\) in parallel to \(QR\) and \(\frac{PM}{MQ} = \frac{2}{3}\)

(i) Find \(\frac{MN}{QR}\)

(ii) Prove that \(\triangle OMN\) and \(\triangle ORQ\) are similar.

(iii) Find the area of \(\triangle OMN\): Area of \(\triangle ORQ\)

Sol. In \(\triangle PQR\), \(MN \parallel QR\) is such a way that \(PM : MQ = 2 : 3\)

(i) In \(\triangle PQR\), \(MN \parallel QR\)

\[
\therefore \frac{PM}{MQ} = \frac{PN}{NR} = \frac{2}{3} \Rightarrow \frac{MQ}{PM} = \frac{3}{2}
\]

Adding 1 on both sides,

\[
1 + \frac{MQ}{PM} = \frac{3}{2} + 1
\]

\[
\Rightarrow \frac{PM + MQ}{PM} = \frac{3 + 2}{2}
\]

\[
\frac{PQ}{PM} = \frac{5}{2} \Rightarrow \frac{PM}{PQ} = \frac{2}{5}
\]

Now in \(\triangle PMN\) and \(\triangle PQR\),

\[
\angle PMN = \angle PQR \quad \text{ (corresponding angles)}
\]

\[
\angle P = \angle P \quad \text{ (Common)}
\]

\[
\therefore \triangle PMN \sim \triangle PQR \quad \text{ (AA postulates)}
\]

\[
\therefore \frac{PM}{PQ} = \frac{MN}{QR} = \frac{PN}{NR}
\]
But \( \frac{PM}{PQ} = \frac{2}{5} \)

\[ \therefore \frac{MN}{QR} = \frac{2}{5} \]

(ii) In \( \triangle OMN \) and \( \triangle ORQ \)

(a) \( \angle MON = \angle QOR \) (Vertically opposite angles)

Since \( MN \parallel QR \),

(b) \( \angle MNO = \angle OQR \) (Alternate angles)

(c) \( \angle NMO = \angle ORQ \) (Alternate angles)

By AAA postulates,

\( \triangle OMN \sim \triangle ORQ \)

(iii) \[
\frac{\text{Ar}(\triangle OMN)}{\text{Ar}(\triangle ORQ)} = \frac{MN^2}{QR^2}
\]

\[
\frac{\text{Ar}(\triangle OMN)}{\text{Ar}(\triangle ORQ)} = \frac{4}{25}
\]

(c) The following figure represents a solid consisting of a right circular cylinder with a hemisphere at one end and a cone at the other. This common radius is 7 cm. The height of the cylinder and cone are each of 4 cm. Find the volume of the solid.

\[ \text{Sol.} \quad \text{Volume} = \text{Volume of cone} + \text{Volume of cylinder} + \text{Volume of hemisphere} \]

\[ = \frac{1}{3} \pi r^2 h + \pi r^2 H + \frac{2}{3} \pi r^3 = \frac{1}{3} \pi r^2 (h + 3H + 2r) \]

\[ = \frac{1}{3} \times \frac{22}{7} \times 7 \times 7 \times 4 + 4 \times 3 + 2 \times 7 \]

\[ = \frac{1}{3} \times 22 \times 7 \times 30 \]

\[ = 22 \times 7 \times 10 \]

\[ = 1540 \text{ cm}^3 \]

Topic: Geometry_Subtopic: Similarity_ Level:2 _Std. X__ICSE Board / Mathematics

Topic: Mensuration_Subtopic: Cylinder_ Level: 2 Std. X__ICSE Board / Mathematics
Question 10

(a) Use Remainder theorem to factorize the following polynomial:

\[ 2x^3 + 3x^2 - 9x - 10 \]

Sol.

\[ p(x) = 2x^3 + 3x^2 - 9x - 10 \]

\[ p(-1) = 2(-1) + 3(1) - 9(-1) - 10 = 0 \]

\[ \therefore x + 1 \] is a factor of \( p(x) \)

Now, dividing \( p(x) \) by \( x + 1 \), we get

\[
\begin{array}{c|ccc}
  & 2x^3 & + x^2 & - 10x \\
\hline
x + 1 & 2x^3 & + 3x^2 & - 9x - 10 \\
      & -2x^3 & - 2x^2 & \\
      &       & x^2 & - 10x - 10 \\
      &       & -x & + x \\
      &       &       & -10x - 10 \\
      &       &       & + x \\
      &       &       & 0 \\
\end{array}
\]

\[ \therefore 2x^3 + 3x^2 - 9x - 10 = (x + 1)(2x^2 + x - 10) \]

\[ = (x + 1)[2x^2 + 5x - 4x - 10] \]

\[ = (x + 1)[x(2x + 5) - 1(2x + 5)] \]

\[ = (x + 1)(x - 2)(2x + 5) \]

Topic: Algebra_Subtopic:Remainder & Factor theorem_ Level:1 _Std. X__ICSE Board / Mathematics

(b) In the figure given below ‘O’ is the center of the circle. If QR = OP and \( \angle ORP = 20^\circ \). Find the value of ‘x’ giving reasons.

\[ \text{Diagram} \]

\[ T \]

\[ O \]

\[ S \]

\[ P \]

\[ Q \]

\[ R \]
(c) The angle of elevation from a point P of the top of a tower QR, 50 m high is 60° and that of the tower PT from a point Q is 30°. Find the height of the tower PT, correct to the nearest metre. [4]
Let the height of the tower PT is \( h \) and PQ is \( x \)

In \( \Delta PQT \)

\[
\tan 30^\circ = \frac{PT}{PQ}
\]

\[
\frac{1}{\sqrt{3}} = \frac{H}{x}
\]

\[x = \sqrt{3} \cdot h\] ....(1)

In \( \Delta PQR \)

\[
\tan 60^\circ = \frac{50}{x}
\]

\[\sqrt{3} \cdot x = 50\] ....(2)

\[\Rightarrow \sqrt{3} \left( \sqrt{3} \cdot h \right) = 50\]

\[3h = 50\]

\[h = \frac{50}{3}\]

\[\therefore h = \frac{50}{3}\]

**Topic: Trigonometry  Subtopic: Heights & Distances  Level: 2  Std. X  ICSE Board / Mathematics**

Question 11

(a) The 4th term of an A. P. is 22 and 15th term is 66. Find the first term and the common difference. Hence find the sum of the series to 8 terms.

**Sol.**
Let the first term at a A.P. is \( a \) and common difference is \( d \).

\[a_4 = a + 3d = 22\] ....(1)

\[a_{15} = a + 14d = 66\] ....(2)

\[\therefore -11d = -44\]

\[a = 4\]

Put \( d = 4 \) in equation (1)

\[a + 3 \times 4 = 22\]

\[a + 12 = 22\]

\[a = 10\]

\[S_n = \frac{n}{2} \left[ 2a + (n-1)d \right]\]

\[S_8 = \frac{8}{2} \left[ 20 + 7 \times 4 \right]\]

\[S_8 = 4 \left[ 20 + 8 \times 4 \right]\]
\[ S_n = 4\left[ 48 \right] = 192 \]

**_topic:** Algebra **_Subtopic:_** A.P. **_Level:_** 1 **_Std._ X **_ICSE Board / Mathematics*

(b) Use graph paper for this question.
A survey regarding height (in cm) of 60 boys belonging to Class 10 of a school was conducted. The following data was recorded:

<table>
<thead>
<tr>
<th>Height in cm</th>
<th>No. of boys</th>
</tr>
</thead>
<tbody>
<tr>
<td>135-140</td>
<td>4</td>
</tr>
<tr>
<td>140-145</td>
<td>8</td>
</tr>
<tr>
<td>145-150</td>
<td>20</td>
</tr>
<tr>
<td>150-155</td>
<td>14</td>
</tr>
<tr>
<td>155-160</td>
<td>7</td>
</tr>
<tr>
<td>160-165</td>
<td>6</td>
</tr>
<tr>
<td>165-170</td>
<td>1</td>
</tr>
</tbody>
</table>

Taking 2cm = height of 10 cm along one axis and 2 cm = 10 boys along the other axis draw an ogive of the above distribution. Use the graph to estimate the following:

(i) the medium
(ii) lower Quartile
(iii) if above 158 cm is considered as the tall boys of the class. Find the number of boys in the class who are tall.

**Topic:** Statistics **_Subtopic:_** Median & Quartiles **_Level:_** 2 **_Std._ X **_ICSE Board / Mathematics*

Sol.

<table>
<thead>
<tr>
<th>Height (in cm)</th>
<th>No.of boys</th>
<th>C.f.</th>
</tr>
</thead>
<tbody>
<tr>
<td>135-140</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>140-145</td>
<td>8</td>
<td>12</td>
</tr>
<tr>
<td>145-150</td>
<td>20</td>
<td>32</td>
</tr>
<tr>
<td>150-155</td>
<td>14</td>
<td>46</td>
</tr>
<tr>
<td>155-160</td>
<td>7</td>
<td>53</td>
</tr>
<tr>
<td>160-165</td>
<td>6</td>
<td>59</td>
</tr>
<tr>
<td>165-170</td>
<td>1</td>
<td>60</td>
</tr>
</tbody>
</table>

\[ \text{Median} = \frac{60}{2} = 30^{th} \text{item} \]
(i) Median = 150.5
(ii) Lower quartile = 142.5
(iii) 10 boys are above 158 cm