SSC - BOARD - 2018

Date: 12.03.2018 **MATHEMATICS - PAPER-2 - SOLUTIONS**

Q.1 Attempt any FIVE of the following sub-questions : **[5]**

(i) $\Delta DEF \sim \Delta MNK$. If $DE = 5$ and $MN = 6$, then find the value of $\frac{A(\Delta DEF)}{A(\Delta MNK)}$.

Ans. $\frac{A(\Delta DEF)}{A(\Delta MNK)} = \frac{DE^2}{MN^2} = \frac{25}{36}$ (Ratio of areas of similar triangles is equal to squares of corresponding sides)

Topic:Similarity_; Sub-topic:Area of similar triangle__L-1__SSC Board Test_Mathematics

(ii) If two circles with radii 8 cm and 3 cm respectively touch externally, then find the distance between their centres.

Ans. $\ell(C_1C_2) = r_1 + r_2 = 8 + 3 = 11\,cm$

If two circles touches externally then distance between their centres is equal to sum of the radii.

Topic:Circle_; Sub-topic:Properties of circle__L-1__SSC Board Test_Mathematics

(iii) Find the length of the altitude of an equilateral triangle with side 6 cm.

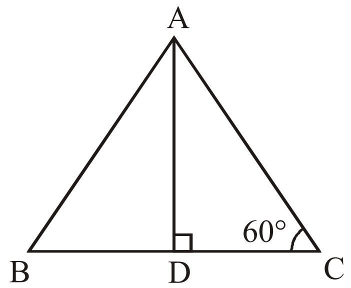
Ans. In ΔADC

$\sin 60° = \frac{AD}{AC}$

$\frac{\sqrt{3}}{2} \times AC = AD$

$\frac{\sqrt{3}}{2} \times 6 = AD$

$3\sqrt{3}\,cm = AD$



OR

Alternate Method :

Altitude of equilateral triangle $= \frac{\sqrt{3}}{2} \times \text{side} = \frac{\sqrt{3}}{2} \times 6 = 3\sqrt{3}\,cm$

Topic:Trigonometry_; Sub-topic:Application of trigonometry__L-2__SSC Board Test_Mathematics

(iv) If $\theta = 45°$, then find $\tan\theta$.

Ans. Given

$\theta = 45°$

$\tan\theta = \tan 45° = 1$

Topic:Trigonometry_; Sub-topic:Angle__L-1__SSC Board Test_Mathematics

(v) Slope of a line is 3 and y intercept is –4. Write the equation of a line.

Ans. Given

Slope of a line $(m) = 3$

y intercept $(c) = -4$

$\therefore$ Equation of line having slope (m) and y-intercept (c) is

$y = mx + c$

$y = 3x - 4$

Topic:Coordinate geometry_; Sub-topic:Equation of line(Slope)__L-1__SSC Board Test_Mathematics

(vi) Using Euler's formula, find V, if $E = 30$, $F = 12$.

Ans. Euler's formula

$F + V = E + 2$

$12 + V = 30 + 2$

$V = 32 - 12 = 20$

Topic:Mensuration_; Sub-topic:Euler's formula__L-1__SSC Board Test_Mathematics

Q.2 Attempt any FOUR of the following subquestions : [8]

(i) The ratio of the areas of two triangles with common base is 6 : 5. Height of the larger triangle of 9 cm, then find the corresponding height of the smaller triangle.

Ans. Let the height of the larger triangle be h_1 and that of the smaller tringle be h_2,

The ratio of the areas of two triangles with common base is equal to the ratio of their corresponding heights.

$$\frac{A(\text{Larger}\,\Delta)}{A(\text{Smaller}\,\Delta)} = \frac{h_1}{h_2}$$

$\therefore \frac{6}{5} = \frac{9}{h_2}$ ...(Substituting the given values)

$\therefore 6 \times h_2 = 9 \times 5$

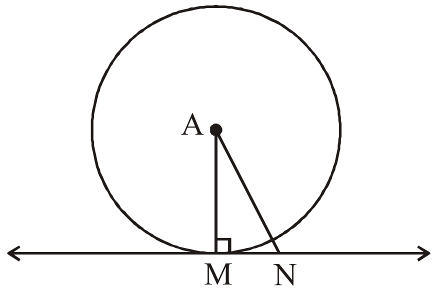
$\therefore h_2 = \frac{9 \times 5}{6} = \frac{15}{2}$

$\therefore h_2 = 7.5\,cm$

The corresponding height of the smaller traingles of 7.5 cm.

Topic:Similarity_; Sub-topic:Area of similar triangle__L-1__SSC Board Test_Mathematics

(ii) In the following figure, point 'A' is the centre of the circle. Line MN is tangent at point M. If $AN = 10$ cm and $MN = 5$ cm, determine radius of the circle.



Ans. Seg AM is the radius and line MN is the tangent to the circle at the point M.

$\therefore\ \angle AMN = 90°$ ...(Tangent is perpendicular to the radius)

In right angled ΔAMN, by Pythagoras' theorem,

$$AN^2 = AM^2 + MN^2$$

$$\therefore\ (10)^2 = AM^2 + (5)^2$$

$$\therefore\ AM^2 = (10)^2 - (5)^2 = 100 - 25 = 75$$

$$\therefore\ AM = \sqrt{75} = \sqrt{25 \times 3}$$

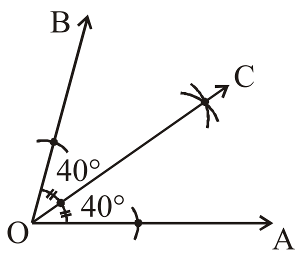
$$\therefore\ AM = 5\sqrt{3}cm$$

The radius of the circle is $5\sqrt{3}cm$.

Topic:Circle_; Sub-topic:Tangent theorem__L-2__SSC Board Test_Mathematics

(iii) Draw $\angle ABC$ of measure 80° and bisect it.

Ans.



Topic:Construction_; Sub-topic:Bisector of angle__L-1__SSC Board Test_Mathematics

(iv) If $\cos\theta = \frac{5}{13}$, where '$\theta$' is an acute angle. Find the value of $\sin\theta$.

Ans. $\cos\theta = \frac{5}{13}$

$$\sin^2\theta = 1 - \cos^2\theta = 1 - \frac{25}{169} = \frac{144}{169}$$

$$\sin\theta = \pm\frac{12}{13}$$

as θ is acute, therefore $\sin\theta$ must be positive.

$$\therefore \sin\theta = \frac{12}{13}$$

Topic:Trigonometry_; Sub-topic:Identities__L-2__SSC Board Test_Mathematics

(v) The volume of a cube is 512 cm^3. Find its side.

Ans. Volume of cube = 512 cm^3

$x^3 = 512$

$x = 8$ cm

$\therefore$ Side is 8 cm.

Topic:Mensuration_; Sub-topic:Volume__L-1__SSC Board Test_Mathematics

(vi) The radius and slant height of a cone are 5 cm and 20 cm respectively. Find its curved surface area. $(\pi = 3.14)$

Ans. $r = 5$ cm, $\ell = 20$ cm

$\therefore$ Curved surface area $= \pi r \ell$

$= 3.14 \times 5 \times 20$

$= 3.14 \times 100$

$= 314\, cm^2$

Topic:Mensuration_; Sub-topic:Volume__L-1__SSC Board Test_Mathematics

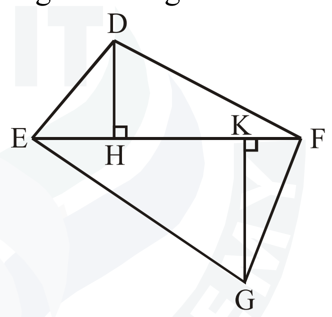
Q.3 Attempt any THREE of the following subquestions : [9]

(i) In the following figure, seg $DH \perp$ seg EF and seg $GK \perp$ seg EF. If DH = 18 cm, GK = 30 cm and $A(\Delta DEF) = 450\, cm^2$, then find :

(i) EF

(ii) $A(\Delta GFF)$

(iii) $A(\square DFGE)$



Ans. (i) Area of a triangle $= \frac{1}{2} \times \text{base} \times \text{height}$

$\therefore A(\Delta EEF) = \frac{1}{2} \times EF \times DH$

$\therefore 450 = \frac{1}{2} \times EF \times 18$ ....(Substituting the given values)

$\therefore \frac{450 \times 2}{18} = EF$

$\therefore EF = 50$ $\therefore EF = 50$ cm

(ii) ΔDEF and ΔGEF ahve the common bae EF.

$\therefore$ their areas are proportional to their corresponding heights.

$\therefore \frac{A(\Delta DEF)}{A(\Delta GEF)} = \frac{DH}{GK}$

$\therefore \frac{450}{A(\Delta GEF)} = \frac{18}{30}$ ....(Substituting the given values)

$\therefore A(\Delta GEF) = \frac{450 \times 30}{18} = 750 cm^2$

$\therefore A(\Delta GEF) = 750$ cm^2

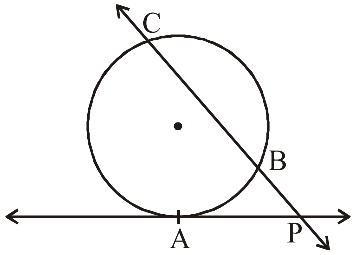
(iii) $A(\square DFGE) = A(\square DEF) + A(\square GEF)$ ....(Area addition postulate)

$= 450 + 750 = 1200\,cm^2$

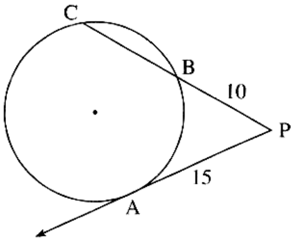
$\therefore A(\square DFGE) = 1200\text{ cm}^2$

Topic:Similarity_; Sub-topic:Area of similar triangle__L-2__SSC Board Test_Mathematics

(ii) In the following figure, ray PA is a tangent to the circle at A and PBC is a secant. If $AP = 15$, $BP = 10$, then find BC.



Ans.



In the given figure, PA is a tangent segment and PBC is the secant.

$\therefore \quad PB \times PC = PA^2$

$\therefore \quad 10 \times PC = 15^2 \qquad \therefore \quad PC = \dfrac{225}{10} = 22.5$

Now, $PB + BC = PC$ .....(P – B – C)

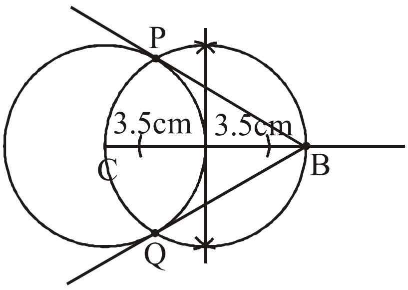
$\therefore \quad 10 + BC = 22.5 \qquad \therefore \quad BC = 22.5 - 10 = 12.5$

Therefore BC = 12.5.

Topic:Circle_; Sub-topic:Secant theorem__L-1__SSC Board Test_Mathematics

(iii) Draw the circle with centre C and radius 3.5 cm. Point B is at a distance 7 cm from the centre. Draw tangents to the circle from the point B.

Ans.



(a) Take centre C
(b) Draw circle of radius 3.5 cm
(c) Extend CA till CB where $\ell(CB) = 7cm$
(d) Take distance more than half in compass and mark two arcs from the ends to line CB.
(e) Now draw two more arc above and below the line keeping compass on these arcs.

Topic:Construction_; Sub-topic:Tangent__L-1__SSC Board Test_Mathematics

(iv) Show that :

$$\sqrt{\frac{1+\cos A}{1-\cos A}} = \text{cosec } A + \cot A$$

Ans. $LHS = \sqrt{\frac{1-\cos A}{1+\cos A}}$

$$= \sqrt{\frac{1-\cos A}{1+\cos A} \times \frac{1-\cos A}{1-\cos A}} = \sqrt{\frac{(1-\cos A)^2}{1-\cos^2 A}}$$

$$= \sqrt{\frac{(1-\cos A)^2}{\sin^2 A}} = \frac{1-\cos A}{\sin A} = \frac{1}{\sin A} - \frac{\cos A}{\sin A} = \text{cosec} A - \cot A = RHS$$

Hence prove.

Topic:Trigonometry_; Sub-topic:Indentities__L-2__SSC Board Test_Mathematics

(v) Write the equation of line passing through $A(-3, 4)$ and $B(4, 5)$ in the form of $ax + by + c = 0$.

Ans. $A(-3,4)\ B(4,5)$

Let $A(-3,4) \equiv (x_1, y_1); B(4,5) \equiv (x_2, y_2)$

Now, $\frac{x - x_1}{x_1 - x_2} = \frac{y - y_1}{y_1 - y_2}$

$\therefore \frac{x-(-3)}{-3-4} = \frac{y-4}{4-5}$ $\qquad$ $\therefore \frac{x+3}{-7} = \frac{y-4}{-1}$

$\therefore -1(x+3) = -7(y-4)$

$\therefore -x-3 = -7y+28$

$\therefore -x+7y-3-28 = 0$ $\qquad$ $\therefore x-7y+31 = 0$

The equation of the line AB is

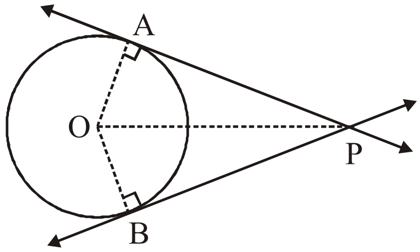
$x - 7y + 31 = 0$

Topic:Coordinate geometry_; Sub-topic:Equation of line__L-1__SSC Board Test_Mathematics

Q.4 Attempt any TWO of the following subquestions : [8]

(i) Prove that, "the lengths of the two tangent segments to a circle drawn from an external point are equal".

Ans.



Given : O is the centre of the circle and P is a point in the exterior of the circle. A and B are the points of contact of the two tangents from P to the circle.

To Prove : $PA = PB$.

Construction : Draw seg OA, seg OB and seg OP.

Proof : Line AP $\perp$ radius OA and line BP $\perp$ radius OB ... (Tangent perpendicular to radius)

$\therefore \angle PAO = \angle PBO = 90°$

In right angled tringles, ΔOAP and ΔOBP,

hypotenuse $OP \cong$ hypotenuse OP ...(Common side)

seg $OA \cong$ seg OB ...(Radii of the same circle)

$\therefore \Delta OAP \cong \Delta OBP$ ...(Hypotenuse-side of theorem)

$\therefore$ seg $PA \cong$ seg PB ...(c.s.c.t.)

$\therefore PA = PB$

Topic:Circle_; Sub-topic:Theorem__L-2__SSC Board Test_Mathematics

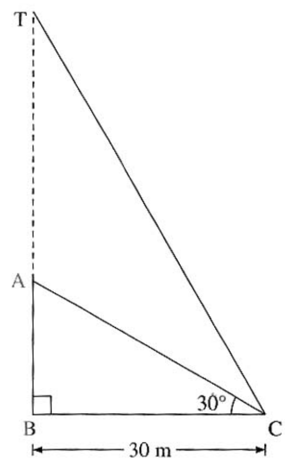
(ii) A tree is broken by the wind. The top of that tree struck the ground at an angle of 30° and at a distance of 30 m from the root. Find the height of the whole tree. $\left(\sqrt{3} = 1.73\right)$

Ans. Let AB represents the unbroken part and AC represent the broken part of the tree. The top of the tree (T) touches the ground at C.

$BC = 30\,cm, \angle ACB = 30°$

Total height of the tree $= AB + AT = AB + AC$ ...(1)

In right angles ΔABC,



$\tan \angle ACB = \frac{AB}{BC}$ $\quad \therefore \tan 30° = \frac{AB}{BC}$

$\therefore \frac{1}{\sqrt{3}} = \frac{AB}{30}$ $\quad \therefore AB = \frac{30}{\sqrt{3}}m$ ...(2)

Also, $\cos \angle ACB = \frac{BC}{AC}$ $\quad \therefore \cos 30° = \frac{BC}{AC}$

$\therefore \frac{\sqrt{3}}{2} = \frac{30}{AC}$ $\quad \therefore AC = 30 \times \frac{2}{\sqrt{3}}$

$\therefore AC = \frac{60}{\sqrt{3}}$ $\quad \therefore AT = \frac{60}{\sqrt{3}}$ ...(3)

Height of the tree $= AB + AT$ ...[From (1)]

$= \frac{30}{\sqrt{3}} + \frac{60}{\sqrt{3}}$ ...[From (2) and (3)]

$= \frac{30+60}{\sqrt{3}} = \frac{90}{\sqrt{3}} = \frac{90}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{90\sqrt{3}}{3}$

$\therefore$ the height of the tree $= 30\sqrt{3}m$

$= 30 \times 1.73\,m = 51.90\,m$

The height of the whole tree is $51.90\,m$.

Topic:Trigonometry_; Sub-topic:App. of trigo__L-3__SSC Board Test_Mathematics

(iii) $A(5, 4)$, $B(-3, -2)$ and $C(1, -8)$ are the vertices of a triangle ABC. Find the equation of median AD.

Ans. Let $A(5,4) \equiv (x_1, y_1); B(-3,-2) \equiv (x_2, y_2)$ and $C(1,-8) \equiv (x_3, y_3)$

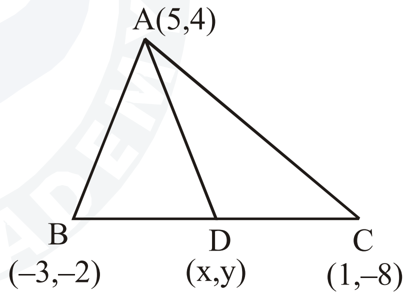
$D(x, y)$ is the midpoint of BC.

$\therefore$ the coordinates of $D = \left(\frac{x_2+x_3}{2}, \frac{y_2+y_3}{2}\right)$

$= \left(\frac{-3+1}{2}, \frac{-2-8}{2}\right) = \left(\frac{-2}{2}, \frac{-10}{2}\right) = (-1, -5)$

Let $D(-1,-5) \equiv (x_4, y_4)$

The equation of median AD is



$\frac{x-x_1}{x_1-x_4} = \frac{y-y_1}{y_1-y_4}$ $\therefore \frac{x-5}{5-(-1)} = \frac{y-4}{4-(-5)}$

$\therefore \frac{x-5}{5+1} = \frac{y-4}{4+5}$ $\therefore \frac{x-5}{6} = \frac{y-4}{9}$

$\therefore \frac{x-5}{2} = \frac{y-4}{3}$

Multiplying both the sides by 6,

$3(x-5) = 2(y-4)$ $\therefore 3x - 15 = 2y - 8$

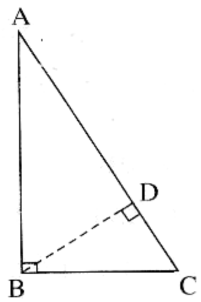
$\therefore 3x - 2y - 15 + 8 = 0$ $\therefore 3x - 2y - 7 = 0$

Topic:Coordinate geometry_; Sub-topic:Equation of line__L-2__SSC Board Test_Mathematics

Q.5 Attempt any TWO of the following subquestions : [10]

(i) Prove that, in a right-angled triangle, the square of hypotenuse is equal to the sum of the square of remaining two sides.

Ans.



Draw perpendicular BD from the vertex B, to the side AC. A – D

In right angled ΔABC,

seg. $BD \perp$ hypotenuse AC.

$\therefore$ by similarity in right angled triangles,

$\Delta ABC \sim \Delta ADB \sim \Delta BDC$

Now, $\Delta ABC \sim \Delta ADB$

$\therefore \frac{AB}{AD} = \frac{AC}{AB}$ ......(c.s.s.t)

$\therefore AB^2 = AC \times AD$ ....(1)

Also, $\Delta ABC \sim \Delta BDC.$

$\therefore \frac{BC}{DC} = \frac{AC}{BC}$ ......(c.s.s.t)

$\therefore BC^2 = AC \times DC$ ....(2)

From (1) and (2),

$AB^2 + BC^2 = AC \times AD + AC \times DC$

$= AC \times (AD + DC)$

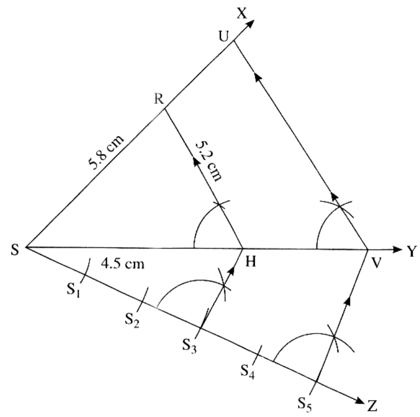
$= AC \times AC$ ....(A – D – C)

$\therefore AB^2 + BC^2 = AC^2$. i.e., $AC^2 = AB^2 + BC^2$

Topic:Similarity_; Sub-topic:Pythagorus theorem__L-2__SSC Board Test_Mathematics

(ii) $\Delta SHR \sim \Delta SVU$, in ΔSHR, $SH = 4.5$ cm, $HR = 5.2$ cm, $SR = 5.8$ cm and $\frac{SH}{SV} = \frac{3}{5}$. Construct ΔSVU.

Ans.



Topic:Construction_; Sub-topic:Construction of triangle__L-3__SSC Board Test_Mathematics

(iii) If 'V' is the volume of a cuboid of dimensions $a \times b \times c$ and 'S' is its surface area, then prove that :

$$\frac{1}{V} = \frac{2}{S}\left[\frac{1}{a} + \frac{1}{b} + \frac{1}{c}\right]$$

Ans. The volume (V) of a cuboid $= a \times b \times c$

$\therefore \frac{1}{V} = \frac{1}{abc}$ ....(1)

The surface area (S) of a cuboid $= 2(ab + bc + ca)$ .....(2)

$RHS = \frac{2}{S}\left(\frac{1}{a} + \frac{1}{b} + \frac{1}{c}\right)$

$= \frac{2}{S}\left(\frac{ab + bc + ca}{abc}\right)$

$$= \frac{2(ab+bc+ca)}{S(abc)}$$

$$= \frac{S}{S(abc)} \qquad \text{....[From (2)]}$$

$$= \frac{1}{abc} = \frac{1}{V} \qquad \text{....[From (1)]}$$

$$\therefore \quad \frac{1}{V} = \frac{2}{S}\left(\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right).$$

Topic:Mensuration_; Sub-topic:Volume_L-3__SSC Board Test_Mathematics