IAPT / ASTROMNY / SOLUTIONS
CODE : A421

1. (B)

$$\frac{GM^2}{(2R)^2} = \frac{MV^2}{R}$$

$$\therefore v = \sqrt{\frac{GM}{4R}}$$

$$T = \frac{2\pi R}{\sqrt{\frac{GM}{4R}}} = 4\pi\sqrt{\frac{R^3}{GM}}$$

$$T.E = -\frac{GM^2}{2R} + 2\left(\frac{GM^2}{8R}\right)$$

$$\therefore \; = -\frac{GM^2}{4R}$$

$$\therefore \text{ Energy} = \frac{GM^2}{4R}$$

2. (B)

If $x = \sqrt[3]{n + \sqrt[3]{n + \ldots\ldots}} \in N$

$\Leftrightarrow x^3 = n + x$

$\Leftrightarrow n = x^3 - x$ and $n \in N,\ n \le 50$ & $x \in N$

$\Leftrightarrow x = 2, 3$

Hence two values of n possible.

3. (C)

The image of monkey moves as fast as that of monkey.

4. (C)

$i = e^{(4n+1)i\frac{\pi}{2}} \Rightarrow i^{2i} = e^{-(4n+1)\pi}$ $n \in I$

Hence , i^{2i} is a real number.

5. (B)

$E_{lost} = mgH(1-0.8) + mgH0.8(1-0.8) + mgH(0.8)^2(1-0.8)$

$= mgH\,0.2\left[1+0.8+(0.8)^2\right]$

$= mgH\,0.2[1+0.8+0.64]$

$= mgH\,0.2\times(2.44)$

$= mgH\,0.488$

$m500\,\Delta T = m10\times 0.488$

$\therefore \Delta T = \dfrac{4.88}{500} = \dfrac{9.76}{1000}$

$\Delta T = 0.00976 = 0.01°C$

6. (B)

Circumradius = R = 65 m

In radius = s – c = 20 m

Let Ram cover m rounds and Shyam cover n rounds. Since their speeds are same, they'll cover same distance

$\Rightarrow 2m\pi \times 20 = 2n\pi \times 65$

$\Rightarrow 4m = 13n$

$\Rightarrow \dfrac{m}{n} = \dfrac{13}{4}$

Hence they'll be close again after 13 rounds by Ram and 4 rounds by shyam

7. (B)

$i^i = e^{-(4n+1)\frac{\pi}{2}} \in R^+$ and $1 \in R^+$

Hence angle between i^i and 1 is 0^0

8. (*)

In diagram A :

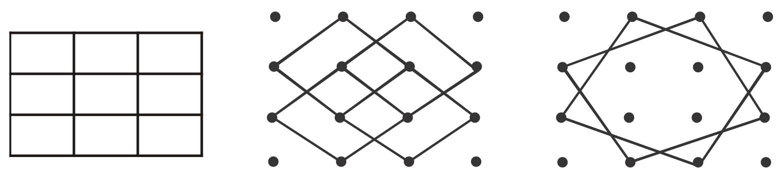
$\because$ no of ways two vertical sides can be selected = ${}^4C_2$

and no of ways two horizontal sides can be selected = ${}^4C_2$

No. of rectangles ${}^4C_2 \cdot {}^4C_2 = 36$

In diagram B : there are 6 rectangles , In diagram C : there are 2 rectangles

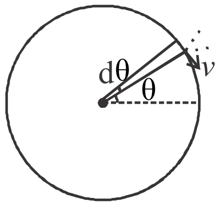
hence total 44 rectangles



(A) (B) (C)

9. (C)

$$dp = \frac{m}{\pi} d\theta\, v$$



$$dm = Rd\theta \frac{m}{\pi R}$$

$$dp_h = 2\frac{m}{\pi} v \sin\theta d\theta$$

$$p_{net} = \frac{2mv}{\pi}\left(-\cos\theta\right)_0^{\pi/2}$$

$$p_{net} = \frac{2mv}{\pi}$$

10 (C)

Any number $x \equiv 0, \pm 1, \pm 2, \pm 3, 4 \pmod 8$

$\Rightarrow x^2 \equiv 0,1,4 \pmod 8$

$\Rightarrow 2a^2 \equiv 0, 2 \pmod 8$

$\Rightarrow b^2 \equiv 0, 1, 4 \pmod 8$

$\Rightarrow 2a^2 + b^2 \equiv 0, 1, 4, 2, 3, 6 \pmod 8$

$\Rightarrow 8c + 7 \equiv 0, 1, 4, 2, 3, 6 \pmod 8$

Which is not possible, because $8c + 7 \equiv 7 \not\equiv 0,1,2,3,4,6$ (mod 8). In other words if we divide $8c + 7$ *by* 8, remainder is 7 but if we divide $2a^2 + b^2$ by 8, remainder can only be 0, 1, 2, 3, 4, 6. Hence not possible.

11. (A)

$t = G^a C^b h^c$

$$\left[M^0 L^0 T^1\right] = \left[\frac{ML^3T^{-2}}{M^2}\right]^a \left[\frac{L}{T}\right]^b \left[ML^2T^{-1}\right]^c$$

$= M^{-a+c} L^{3a+b+2c} T^{-2a-b-c}$

$-a + c = 0 \quad \therefore a = c$

$3a + b + 2a = 0$

$b = -5a$

$-2a - b - c = 1$

$-2a + 5a - a = 1$

$\therefore 2a = 1 \qquad \therefore a = \frac{1}{2}$

$b = \frac{-5}{2}$

$$\left[G^{1/2} C^{-5/2} h^{1/2}\right]$$

12. (B)

$P(0) = P(-1) = P(-2) = ... = P(-2001)$

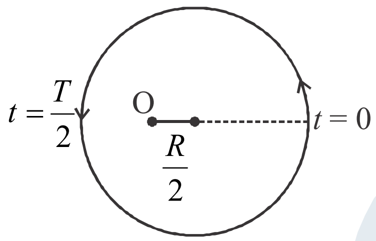
$\Rightarrow$ by Rolle's theorem $P'(x) = 0$ has atleast 2001 distinct roots lying in

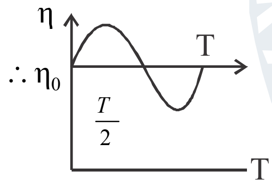
$(-2001, -2000), (-2000, 1999), . . , (-1, 0)$

Since $P(x)$ is a polynomial of degree 2002, $P'(x)$ has degree.2001, therefore $P'(x)$ cannot have more than 2001 roots

$\Rightarrow P'(x) = 0$ has exactly 2001 distinct roots. Now observe if $P(x) = 0$ has root of multiplicity k then $P'(x) = 0$ will have same root of multiplicity $k-1$. Since no root of $P'(x) = 0$ is repeated, we must have $k - 1 \le 1 \Rightarrow k \le 2 \Rightarrow$ max multiplicty of roots of $P(x) = 0$ is 2.

13. (B)





14 (C)

$$\int_0^{\sqrt{2}} \{x^2\}\, dx = \int_0^1 \{x^2\} dx + \int_1^{\sqrt{2}} \{x^2\}\, dx$$

$$= \int_6^1 x^2\, dx + \int_1^{\sqrt{2}} (x^2 - 1)\, dx = \frac{1}{3} + \frac{2\sqrt{2}}{3} - \frac{1}{3} - (\sqrt{2} - 1)$$

$$= 1 - \frac{\sqrt{2}}{3}$$

15. (D)

$F_b = 7.5 - 2.5 = 5\, kg\, wt$

$$\rho = \frac{F_b}{V_g} = \frac{50}{3 \times 10^{-3} \times 10} = \frac{5000}{3} kg/m^3$$

$W = F_b + T = 5 + 2.5 = 7.5\, kg\, wt.$

16. (C)

Take c = a in a AM–HM inequality

$$\Rightarrow \frac{2a+b}{3} \geq \frac{3a^2b}{ab+ab+a^2} \Leftrightarrow \frac{9ab}{2b+a} \leq 2a+b$$

Hence option (C) is correct.

It can be verified that options (A), (B) (D) are wrong by counter example $a = b$

17. (A)

The formation of rainbow is based on T.I.R., Dispersion and deviation

18. (D)

$f(x) = \cos x + \cos(\pi x)$ is not periodic $\Rightarrow$ A is false

$f(x) = |\cos x| + |\sin x|$ is periodic with period $= \frac{\pi}{2}$ $\neq$ LCM of periods of $|\cos x|$ and $|\sin x|$

$\Rightarrow$ Hence B is also false.

19. (D)

$$R = \frac{1+V_0}{40} = \frac{3+V_0}{80}$$

$$\therefore\ 2+2V_0 = 3+V_0$$

$$\therefore\ V_0 = 1V$$

$$R = \frac{2}{40} \times 10^3 = \frac{100}{2} = 50\Omega$$

20. (D)

$$f(x) = \sin x + \cos x \qquad \Rightarrow y = \sqrt{2}\sin\left(x+\frac{\pi}{4}\right)$$

$$\Rightarrow x = \sin^{-1}\left(\frac{y}{\sqrt{2}}\right) - \frac{\pi}{4} \qquad \Rightarrow f^{-1}(x) = \sin^{-1}\left(\frac{x}{\sqrt{2}}\right) - \frac{\pi}{4}$$

21. (B)

$$m_A u = m_B v - m_A v$$

$$\therefore 1 = \frac{2v}{u}$$

$$\therefore u = 2v$$

$$\therefore 2m_A = m_B - m_A$$

$$3m_A = m_B$$

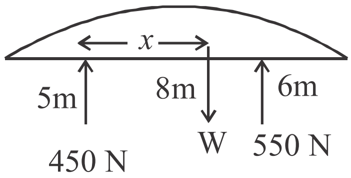
$$\frac{m_A}{m_B} = \frac{1}{3}$$

22. (D)

$$\lim_{x \to -\frac{\pi}{4}} \frac{\sin x + \cos x}{x + \frac{\pi}{4}} = \lim_{x \to -\frac{\pi}{4}} \frac{\sqrt{2}\sin\left(x + \frac{\pi}{4}\right)}{x + \frac{\pi}{4}}$$

$$= \sqrt{2} \lim_{\theta \to 0} \frac{\sin\theta}{\theta} = \sqrt{2}$$

23. (B)



$\therefore W = 450 + 550 = 1000\,N$

$m = 100\,kg$

$wx = 550 \times 8$

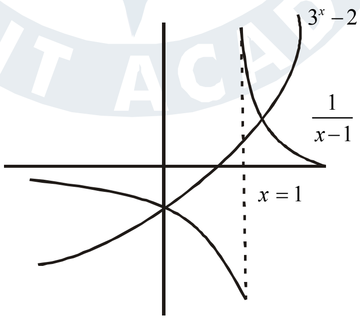
$1000x = 550 \times 8$

$x = \frac{55 \times 8}{100} = \frac{440}{100} = 4.4\,m$

24. (C)

$$3^x - 2 = \frac{1}{x-1}$$

Two intersection graphically



Proof: If $x < 1$ the funciton $3^x - 2 - \frac{1}{x-1}$ is an incrasing funciton $\Rightarrow$ max one root. Since $x = 0$ is a root $\Rightarrow$ exactly one root for $x < 1$. Similarly if $x > 1$, the funciton $3^x - 2 - \frac{1}{x-1} = f(x)$ is an increasing funciton $\Rightarrow$ max one root.Now

$f(1^+) = -\infty$ and $f(\infty) = \infty$

$\Rightarrow$ minimum one root by intermediate value theorem. Hence one root for $x > 1$. Hence total 2 roots.

25. (A)

$$\frac{1}{v}+\frac{1}{u}=\frac{1}{f} \qquad \frac{1}{v}-\frac{1}{2.8}=\frac{1}{0.2}$$

$$\therefore \frac{1}{v}=\frac{1+14}{2.8}=\frac{15}{2.8}$$

$$v=\frac{2.8}{15}m$$

$$\frac{dv}{dt}=-\left(\frac{v}{u}\right)^2\frac{du}{dt}=-\left(\frac{2.8}{15\times 2.8}\right)^2\times 15=-\frac{1}{15}m/s$$

26. (D)

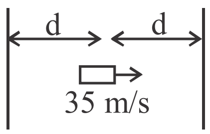
$$\log_{\sqrt{2}}16=8$$

$$\log_{27}9=\frac{2}{3}$$

$$\log_{1/3}3=-1$$

$$\Rightarrow \text{Ans}=\frac{23}{3}$$

27. (B)



wall 1

$$t_1'=\frac{d}{350} \qquad \therefore t_1''=\frac{d-\frac{d}{10}}{350+35}$$

$$t_1=\frac{d}{350}+\frac{9d}{10(35)(11)}$$

wall 2

$$t_2'=\frac{d}{350} \qquad t_2''=\frac{d+\frac{d}{10}}{350-35}$$

$$t_2=\frac{d}{350}+\frac{11d}{10\times 35\times 9}$$

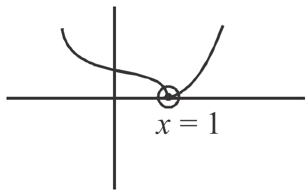
$$\therefore t_2-t_1=1 \qquad \therefore \frac{11d}{35\times 10\times 9}-\frac{9d}{35\times 10\times 11}=1$$

$$\therefore \frac{d}{350}\frac{40}{99}=1 \quad \therefore d=\frac{99\times 35}{4}$$

$$d=866.25m$$

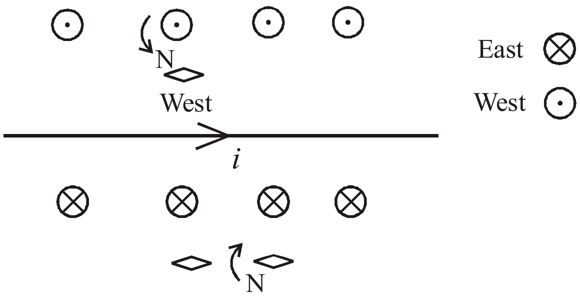
28. (C)

Graph of $\left|x^3-1\right|$ is



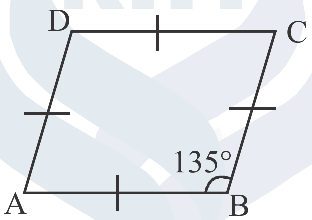
Hence one point of non differentiability which is $x = 1$.

29. (C)



30. (D)

ABCD is a rhombus with angles 45° and 135°



$$\Rightarrow Area = \left|\overline{AB}\times\overline{BC}\right| = \frac{1}{\sqrt{2}}$$

Option C is false as max area of quadrilateral with perimeter 4 = 1 (Area of square)

31. (B)

Conceptual

32. (C)

$$3^8\left(3^{10}+6^5\right)+2^3\left(2^{12}+6^7\right)=3^{18}+3^{13}\cdot 2^5+3^7\cdot 2^{10}+2^{15}$$

$$=\left(3^6+2^5\right)^3 \text{ hence perfect cube}$$

Since $3^6+2^5=761 \neq$ perfect square $\Rightarrow \left(3^6+2^5\right)^3$ cannot be perfect sqaure

33. (A)

$$82\times 3.5\times 10^3\times 2 = N\left(250\times 10^{-6}\times 1000\right)2.3\times 10^6$$

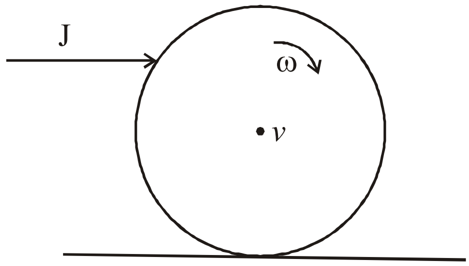
$$N=\frac{82\times 3.5\times 10^3}{25\times 2.3\times 10^4} \qquad N=\frac{574}{575}=1$$

34. (A)

$n = 1 + 12 + 60 + 160 + 240 + 192 + 64$

$= (1+2)^6 \Rightarrow$ perfect square and perfect cube.

35. (*)



$J = mv = (200 \times 10^{-3})(8 \times 10^{-2}\, m/\sec)$

$J = 16 \times 10^{-3}\, kgm/\sec$

$J(\text{perpendicular distance}) = I\omega$

$16 \times 10^{-3}(4 \times 10^{-2}) = \frac{2}{3}(200 \times 10^{-3})(16 \times 10^{-2})^2 \omega$

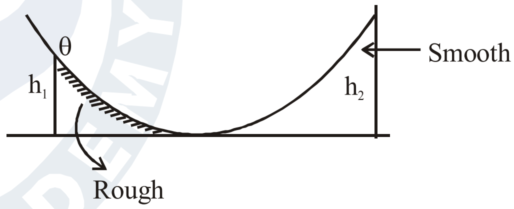
$16 \times 4 \times 10^{-5} = \frac{4}{3} \times 16 \times 10^{-5}\, \omega \times 16$

$\omega = \frac{3}{16}\, rad/\sec$

36. (D)

Conceptual

37. (D)



$mgh_1 = \frac{1}{2} I\omega^2 + \frac{1}{2} mv^2 = \frac{1}{2}\left(\frac{2}{5} mv^2\right)\omega^2 + \frac{1}{2} mv^2$

$mgh_1 = \frac{7}{10} mv^2$

$\frac{5mgh_1}{7} = \frac{1}{2} mv^2$ ....(1)

As the solid sphere ascends on smooth surface there would be no change in angular velocity. So

At heighest point energy is

$= mgh_2 + \frac{1}{2} I\omega^2$

so $mgh_1 = mgh_2 + \frac{1}{2}\left(\frac{2}{5} mr^2\right)\omega^2$

$mgh_1 = mgh_2 + \frac{1}{5} mv^2$

$mgh_1 = mgh_2 + \frac{1}{5}\left(\frac{10\, mgh_1}{7}\right)$

$mgh_2 = mgh_1 - \frac{2\, mgh_1}{7} = \frac{5\, mgh}{7}$

$h_2 = \frac{5h_1}{7} = 50\ cm$

38. (C)

$$3^{12}+2^9+3.\left(3.6^4+6^5\right)+2^6=3^{12}+2^9+3.\left(3^5.2^4+3^5 2^5\right)+2^6$$
$$=3^{12}+2^9+3.3^5.2^4.3+2^6$$
$$=3^{12}+2^9+3^7.2^4+2^6$$
$$=3^{12}+2^6\left(2^3+1\right)+3^7.2^4$$
$$=3^{12}+3^2.2^6+3^7.2^4$$
$$=3^2\left(3^{10}+2^6+3^5.2^4\right)$$
$$=3^2\left(3^4+2^3\right)^2=\left(3^6+3.2^3\right)^2=753^2$$

Hence its a perfect square but not perfect cube as 753 is not a cube.

39. (A)

Torque acting on the body is

$$\tau = fR=\left(40t-10t^2\right)0.1$$
$$\tau = 4t-t^2$$

angular acceleration is $\alpha = \frac{\tau}{1} = \frac{4t - t^2}{1}$

$$\alpha = 4t-t^2$$
$$\frac{d\omega}{dt} = 4t-t^2$$
$$\int d\omega = \int\left(4t-t^2\right)dt$$
$$\omega = \frac{4t^2}{2}-\frac{t^3}{3}$$
$$\omega = 2t^2-\frac{t^3}{3}$$

To get angular velocity reversed, $\omega < 0$

$$2t^2-\frac{t^3}{3}<0$$
$$2t^2<\frac{t^3}{3}$$
$$6<t$$
$$t<6 \text{ sec}$$

40. (B)

$$\log_{10}0.01+\log_{0.1}10+\log_{10}0.001+\log_{0.1}0.001$$
$$=-2-1-3+3=-3$$
$$=\log_{10}0.000001+3$$

41. (A)

9 division of M.S = 10 divisions of V.S.

$$1 \text{ div of V.S.} = \frac{9}{10} \text{ M.S.}$$

$$\text{diff} = 1\text{M.S} - 1\text{V.S} = \frac{1}{10} \text{ M.S} = 0.1\ mm$$

So $x = 3 \times 0.1 = 0.3\ mm$

$x = 0.03\ cm$

42. (C)

Def^n of $A \times B$

43. (A)

$$\text{magnification } m = \frac{f}{f-u}$$

$$m = \frac{f}{f+u}$$

$$m = \frac{f}{f+(u+f)}$$

$$m = \frac{f}{u+2f}$$

$$\frac{f}{f+u} = -\frac{f}{u+2f}$$

$$f + u = -(u+2f)$$

$$2u = -3f$$

$$u = -\frac{3f}{2}$$

$$m = \frac{f}{f - \frac{3f}{2}} = \frac{f}{\frac{f}{2}} = 2$$

$$= \frac{\text{size of image}}{\text{size of object}}$$

image size = 10 cm

44. (D)

Trivial

45. (A)

Due to centrifugal force (observer in Non inertial frame), the string will incline towards right.

46. (C)
Trivial (Note that it is also needed for every element in A to have an image.)

47. (B)
Due to steel ball, more volume would be displace to componsate the weight of the steel ball. Once ice cube is melted, water level will fall down.

48. (C)
C is a definitely false statement where as for the rest truth value cannot be determined uniquely.

49. (A)
Maximum static friciton act on the body is

$f_{\max} = \mu_s N = 0.5(50) = 25\,N$

$f_{applied} < f_{\max}$, so block does not slide

frictional force = applied force.

50. (D)

$S_1 = A \times B, S_2 = A^c \times B, S_3 = A \times B^c, S_4 = A^c \times B^c$

observe that S_1, S_2, S_3, S_4 are all mutually exclusive and exhaustive hence

$(A \times B)^c = (A^c \times B) \cup (A \times B^c) \cup (A^c \times B^c)$

51. (C)
For, bullet coming to halt,

$v^2 - u^2 = 2as$

$0^2 - (72)^2 = 2a(9\,cm)$ ....(1)

If which block is of 8 cm

$v^2 - u^2 = 2as$

$v^2 - (72)^2 = 2a(8\,cm)$ ....(2)

(1) / (2) $\Rightarrow \dfrac{-(72)^2}{v^2 - (72)^2} = \dfrac{9}{8}$

$-8(72)^2 = 9v^2 - 9(72)^2$

$9v^2 = 1(72)^2$

$v^2 = \dfrac{(72)^2}{9} \Rightarrow v_2 = \dfrac{72}{3} = 24\,m/\text{sec}.$

52. (D)
Definition of hyperbola

53. (D)
Loss of weight in water = 12.9 – 11.3 = 1.6 grams × g

$$f_B = (1.6\,gm)\,g$$

$$V \cdot \rho_w \cdot \not{g} = (1.6\,gm)\,g$$

$$(V_c + V_z)(1\,gm/cc) = 16\,gm$$

$$V_c + V_z = 1.6\ cc$$

$$\frac{m_c}{\rho_c} + \frac{m_z}{\rho_z} = 1.6$$

$$\frac{m_c}{8.9} + \frac{12.9 - m_c}{7.1} = 1.6$$

$$m_c\left(\frac{1}{8.9} - \frac{1}{7.1}\right) = 1.6 - \frac{12.9}{7.1}$$

$$m_c = \left(\frac{-1.8}{8.9 \times \not{7.1}}\right) = \frac{1.6 \times 7.1 - 12.9}{\not{7.1}}$$

$$m_c = \frac{(12.9 - 1.6 \times 7.1)8.9}{1.8}$$

$$m_c = \frac{1.54 \times 8.9}{1.8}$$

$$m_c = 7.6\ gm$$

54. (B)
Trivial

55. (C)
Distance travelled by the block on the inclined is

$$l = \frac{h}{\sin 30} = \frac{10}{\left(\frac{1}{2}\right)} = 20\,cm$$

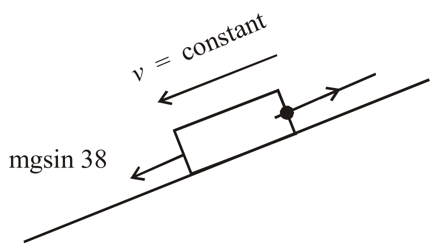
pulley A has to be raised by 20 cm, distance lowered is 40 cm.

56. (B)

$$T = mg\sin 38$$

$$T = 50 \times 10 \times \frac{1}{2} = 250\ N$$

so $f = \frac{\pi}{2} = \frac{250}{2} = 125\ N$



57. (D)

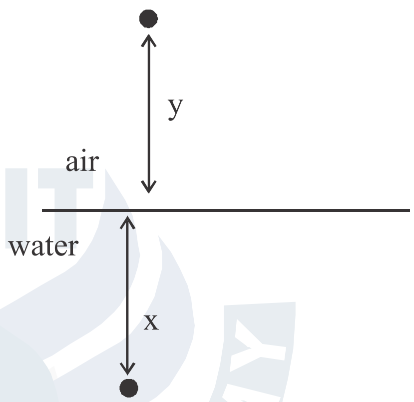
$$\cos^2 x = \frac{1}{3} \Rightarrow \sin^2 x = \frac{2}{3} \Rightarrow \text{cosec } x = \pm\sqrt{\frac{3}{2}}$$

58. (C)

$3^{4k} \equiv 1 \pmod{80}$

$\frac{3^{4k}-1}{10} = 8m$ which is an even number hence second last digit of 3^{4k} is last digit of $\frac{3^{4k}-1}{10}$ which is even similarly we can prove second last digit of 3^n is even for n = 4k + 1 , 4k + 2 , 4k + 3 hence option C is correct. Also observed second last digit of 3^{15} is zero.

59. (*)



$$\frac{x}{\mu} + y = 15$$

$$\frac{3x}{4} + y = 15 \Rightarrow 3x + 4y = 60$$

$$x + \mu y = 20 \Rightarrow x + \frac{4}{3}y = 20$$

$$3x + 4y = 60$$

$$3(x+y) + y = 60$$

$$x + y = 20 - \frac{y}{3} < 20$$

Note : No answer is matching

60. (B)

$$\sqrt{n+1} - \sqrt{n} \le 0.01$$

$$\Leftrightarrow \sqrt{n+1} + \sqrt{n} \ge 100$$

if $n \le 2499$

$$\Rightarrow \sqrt{n+1} + \sqrt{n} \le 50 + \sqrt{2499} < 100$$

if $n \ge 2500$

$$\sqrt{n+1} + \sqrt{n} \ge \sqrt{2501} + 50 > 100$$

$$\Rightarrow n \ge 2500$$

61. (C)

Equivalent resistance $= \frac{2R}{3} = 2.4,$

where R is resistance of each side.

$\therefore R = 3.6\Omega$

or $1 \times x = 3.6$

or $x = 3.6m$

62. (C)

Maximum product will occur when PA = PB = PC = circumradius = $\frac{1}{\sqrt{3}}$

63. (A)

$\square OACB, \square OAC_1B_1$ and $\square OAC_2B_2$ are parallelograms.

$OA = P,\ \ OB = Q,\ \ OB_1 = 2Q,\ \ OB_2 = Q_2$

$OC = R,\ \ OC_1 = 2R,\ \ OC_2 = 2R$

$\therefore\ \ R^2 = P^2 + Q^2 + 2PQ\cos\theta$ ....(1)

$AR^2 = P^2 + 4Q^2 + 4PQ\cos\theta$ ....(2)

$AR^2 = P^2 + Q^2 - 2PQ\cos\theta$ ....(3)

Hence,

$5R^2 = P^2 + Q^2,$

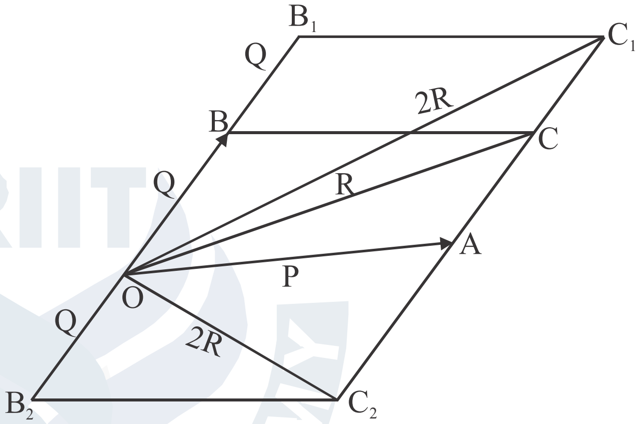
$4R^2 = P^2 + 2Q^2,$ and

$2R^2 = 2Q^2 - P^2.$

$\therefore \sqrt{3}R = \sqrt{2}Q$

and $R = P$

$\therefore P:Q:R = P:\frac{\sqrt{3}P}{\sqrt{2}}:P = \sqrt{2}:\sqrt{3}:\sqrt{2}$



64. (A)

$23^{2015}.7^{2016}.13^{2017}$

$\equiv 3^{2015}.(1)^{504}.3^{2017}$

$\equiv 3^{4032} \equiv 1(\text{mod}10)$

65. (B)

$4\pi r^2 dr\rho = dM$

66. (C)

$\frac{dP}{dr} = -\frac{GM_r}{r^2}\rho_r$

67. (D)

$\frac{O - P_c}{R_0 - O} = -\frac{GM_0 \times 4}{2 \times R_0^2} \times \frac{M_0}{\frac{4}{3}\pi R_0^3}$

$\therefore P_C = \frac{3}{2} \times \frac{GM_0{}^2}{\pi R_0{}^4}$

68. (C)

$$P_C = \frac{3}{2} \times \frac{6.67 \times 10^{-11} \times 4 \times 10^{60}}{\pi \times \left(7 \times 10^{8}\right)^{4}}$$

$$\simeq 5 \times 10^{14}\ N - m^{-2}$$

69. (C)

$$Pv = nRT \ \Rightarrow P = \frac{\rho RT}{M}$$

$$\Rightarrow P = \frac{\rho RT}{M_H N_A} = \frac{\rho kT}{M_H}$$

$$\Rightarrow T = \frac{P}{k} \times \frac{2}{\text{number of particle density}}$$

$$= \frac{5 \times 10^{14}}{1.4 \times 10^{-23}} \times \frac{2}{1.67 \times 10^{27}} \qquad \simeq 4 \times 10^{7}\ k$$

70. (A)

$$m \times r \times \frac{4\pi^2}{T^2} = \frac{GMm}{r^2}$$

$$\text{or} \quad T^2 = \frac{4\pi^2 r^3}{GM} \qquad \text{or} \quad r^3 = \frac{GMT^2}{4\pi^2}$$

$$= \frac{6.67 \times 10^{-11} \times 6 \times 10^{24} \times 144 \times (3600)^2}{4 \times \pi^2}$$

$$r^3 = 1.9 \times 10^{9} \times 10^{13}\ m$$

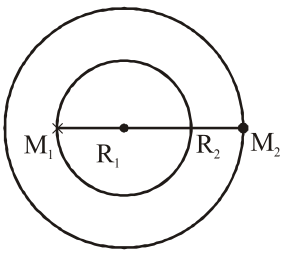
$$r = 1.82 \times 10^{7}\ m$$

71. (A)

$$R_1 = \frac{M_2 d}{M_1 + M_2}$$

$$R_2 = \frac{M_1 d}{M_1 + M_2}$$

$$\therefore \frac{R_1}{M_2} = \frac{R_2}{M_1} \Rightarrow \frac{M_2}{M_1} = \frac{R_1}{R_2}$$



72. (C)

$$M_1R_1 \times \frac{4\pi^2}{T^2} = \frac{GM_1M_2}{\left(R_1 + R_2\right)^2}$$

$$\Rightarrow T^2 = \frac{4\pi^2 R_1\left(R_1+R_2\right)^2}{GM_2}$$

$$T^2 = \frac{4\pi^2 R_2\left(R_1+R_2\right)^2}{GM_1}$$

73. (D)

$$V_1 = 36\,km/s \quad V_2 = 12\,km/s$$

$$\frac{G\,M_1\,M_2}{\left(R_1+R_2\right)^2} = \frac{M_1V_1^2}{R_1}$$

$$W = \frac{V_1}{R_1} = \frac{V_2}{R_2} \qquad \therefore \frac{R_1}{R_2} = \frac{3}{1}$$

$$M_1R_1 = M_2R_2 \qquad \therefore \frac{M_1}{M_2} = \frac{1}{3}$$

$$\therefore \frac{G\,M_1\left(M_1 3\right)}{\left(R_1 + \frac{R_1}{3}\right)^2} = \frac{M_1V_1^2}{R_1}$$

$$\therefore \frac{G3M_1}{\frac{16}{9}R_1} = V_1^2$$

$$\frac{27G}{16}\left(36000\right)^2$$

$$\frac{2\pi R_1}{36000} = 137\times24\times3600$$

$$\therefore R_1 = \frac{13.7\times2.4\times3.6\times3.6\times10^5}{2\pi}$$

$$M_1 = \frac{\left(36\right)^2\times10^6\times16}{27\times6.67\times10^{-11}}\times\frac{137\times24\times36\times36}{2\pi}\times10^5$$

$$M_1 = 7.8\times10^{30}\,kg$$

$$M_2 = 2.3\times10^{30}\,kg$$

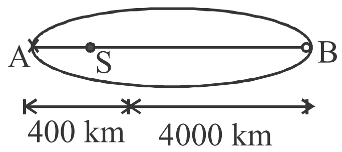
74. (A)

$a = 2200\,km$

$T^2 = \frac{4\pi^2}{GM} a^3$

$T^2 = \frac{4 \times 10 \times \left(22 \times 10^5\right)^3}{6.67 \times 10^{-4} \times 6 \times 10^{24}}$

$T^2 = \frac{4 \times (22)^3 \times 10^{16}}{6.67 \times 6 \times 10^{13}}$



$T^2 = 1064 \times 10^3$

$T^2 = 106.4 \times 10^4$

$T = \sqrt{106.4} \times 10^2$

$T \approx 10 \times 10^2 \approx 10^3$ sec

$T \simeq \frac{1000}{60 \times 60} = \frac{10}{36} \approx 0.29\,hr$

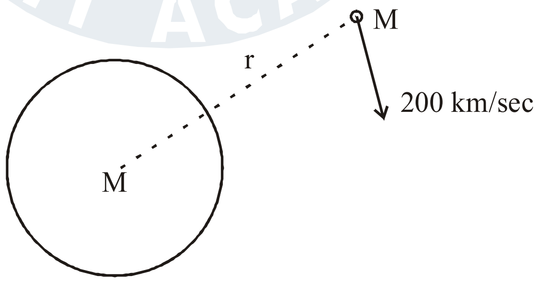
75. (A)

$V_1 R_1 = V_2 R_2$

$\Rightarrow \frac{V_1}{V_2} = \frac{R_2}{R_1} = \frac{4000}{400} = 10:1$

76. (*)

77. (B)



$\frac{GMm}{r^2} = \frac{mv^2}{r}$

$v^2 = \frac{GM}{r}$

$M = \frac{v^2}{G}$

$M = \frac{\left(200 \times 10^3\right)^2 \cdot r}{6.67 + 10^{-11}}$

$M = \frac{4 \times 10^{10} \times 1.6 \times 10^{21}}{6.67 \times 0^{-11}}$

$M = \frac{6.4}{6.67} \times 10^{42}$

$M = 0.95 \times 10^{42}$

$M = 9.5 \times 10^{41}\ kg$

total mass $= 11.5 \times 10^{41}\ kg$

% of dark matter is

$= \frac{9.5 \times 10^{41}}{11.5 \times 0^{41}} \times 100$

$= 82\,\%$

78. (B)

escape velocity is $v_e = \sqrt{\frac{2GM}{R}}$

$$v_e = \sqrt{\frac{2G\left(\rho \cdot \frac{4}{3}\pi R^3\right)}{R}}$$

$$v_e = \sqrt{\frac{8G\rho\pi R^2}{3}}$$

$$v_e \propto \sqrt{\rho R^2}$$

$$\frac{(v_e)_5}{(v_e)_c} = \sqrt{\frac{\rho_j}{\rho_e} \cdot \frac{R_j^2}{R_e^2}}$$

$$\frac{59.5}{11.2} = \sqrt{\frac{\rho_j}{\rho_e}(12)^2}$$

$$5.31 = 12\sqrt{\frac{\rho_i}{\rho_e}}$$

$$\sqrt{\frac{\rho_i}{\rho_e}} = \frac{5.31}{12} = 0.44$$

$$\frac{\rho_j}{\rho_e} = (0.44)^2 \approx 0.19$$

$$\boxed{\rho_j = 0.2\,\rho_e}$$

79. (C)

$$a(1+0.2) = 69$$

$$\therefore a = \frac{69}{1.2}$$

$$\text{Min.distance} = a(1-e) = \frac{69}{1.2} \times 0.8$$

$$= 46 \text{ million km}$$

80. (D)

Pole star cannot be seen from southern hemisphere.