Rao IIT Academy
Symbol of Excellence and Perfection
JEE | MEDICAL-UG | BOARDS | KVPY | NTSE | OLYMPIADS | MHT-CET

SOF - NSO - STAGE-1_12TH STD

ANSWER SHEET

1.	(D)	2.	(B)	3.	(C)	4.	(B)	5.	(A)	6.	(C)
7.	(D)	8.	(D)	9.	(D)	10.	(C)	11.	(B)	12.	(B)
13.	(C)	14.	(B)	15.	(C)	16.	(A)	17.	(D)	18.	(A)
19.	(D)	20.	(D)	21.	(C)	22.	(A)	23.	(A)	24.	(A)
25.	(B)	26.	(B)	27.	(C)	28.	(*)	29.	(B)	30.	(C)
31.	(C)	32.	(A)	33.	(B)	34.	(C)	35.	(D)	36.	(A)
37.	(A)	38.	(D)	39.	(A)	40.	(B)	41.	(B)	42.	(D)
43.	(C)	44.	(A)	45.	(C)	46.	(B)	47.	(A)	48.	(C)
49.	(B)	50.	(A)								

1. **(D)**

$R = 100\,\Omega, V_{rms} = 200\,V, w = 300\,rad/s$. When only capacitor is removed, it becomes LR circuit.

$\therefore \tan\theta = \frac{X_L}{R}$, where $\theta_1 = 60^0$ is the angle of lag for current.

$\therefore \sqrt{3} = \frac{X_L}{R} \Rightarrow X_L = \sqrt{3}\,R$ ......(1)

When only inductor is removed, it becomes RC circuit.

$\therefore \tan\theta_2 = \frac{X_C}{R}$ ($\theta_2 = 60^0$ is the angle by which current leads)

$\therefore \sqrt{3} = \frac{X_C}{R} \Rightarrow X_C = \sqrt{3}\,R$ .....(2)

$\therefore$ In LCR circuit, net reactance, $X = X_C - X_L = \sqrt{3}\,R - \sqrt{3}\,R = 0$

{Using equation (1) and (2)}

$\therefore$ If is equivalent to a purely resistive circuit (condition of resonanece)

$\therefore$ Power dissipitated, $P = \frac{V_{rms}^2}{R} = \frac{(200V)^2}{100\,\Omega} = 400\,W$

2. **(B)**

Effective value of acceleration due to gravity inside the liquid, is

$g_{eff} = \frac{mg - B}{m}$ {B is buoyant force}

$= g - \frac{B}{m} = g - \frac{\rho_l . V . g}{\rho_b\, V}$

{V is volume of the ball, ρ_l and ρ_b are densities of liquid and the ball respectively}

$\Rightarrow g_{eff.} = g - \frac{1}{3}g$ $(\because \rho_b = 3\rho_l)$

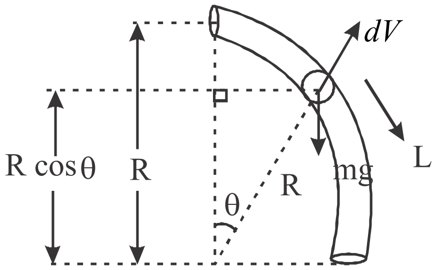
$\Rightarrow g_{eff.} = \frac{2g}{3}$

$\therefore$ time period of oscillations (neglecting the resistance of the liquid) is

$$T = 2\pi\sqrt{\frac{l}{g_{eff.}}} = 2\times 3.142 \times \sqrt{\frac{0.20\,m}{\frac{2}{3}\times 9.8\,m/s^2}}$$

$\Rightarrow T \approx 1.1\,s$

3. (C)



Let dV be the normal reaction by the lower surface when the angle made by radius vector with vertical is θ (assuming contact with lower surface).

In this position, we get

$$\frac{1}{2}mv^2 = mg\ R(1-\cos\theta)$$

{gain is K.E. = los in gravitational P.E.}

$\Rightarrow V^2 = 2gR(1-\cos\theta)$ ....(1)

Now, for circular motion of ball,

net centripetal force $=\frac{mv^2}{R}$

or, $mg\cos\theta - v = \frac{mv^2}{R}$

or, $dV = mg\cos\theta - \frac{mv^2}{R} = mg\cos\theta - 2mg(1-\cos\theta)$

{using equation (1)}

or, $dV = 3mg\cos\theta - 2mg$

$\therefore$ dV is positive for lower values of θ which shows that ball will be initially in contact with lower surface.

$\therefore$ for the point of loosing contact with lower surface,

$dV = 0$

or, $3mg\cos\theta - 2mg = 0$

$\Rightarrow \cos\theta = \frac{2}{3} \Rightarrow \theta = \cos^{-1}\frac{2}{3}$ which is acute.

$\therefore$ later the ball will lose contact with lower surface and come in contact with upper surface.

4. (B)

$T = T_0 + \alpha V^2$ ....(1)

Also, PV = RT (for one mole)

$\Rightarrow PV = R(T_0 + \alpha V^2)$ {using equation (1)}

$\Rightarrow P = R(T_0 V^{-1} + \alpha V)$ ....(2)

$\therefore \quad \frac{dP}{dV} = R(-T_0.V^{-2} + \alpha)$ .....(3)

$\therefore$ for minimum value of P, $\frac{dP}{dV} = 0$

$\Rightarrow \alpha - T_0 V^{-2} = 0 \Rightarrow V^2 = \frac{T_0}{\alpha} \Rightarrow V = \sqrt{\frac{T_0}{\alpha}}$

Now, from eq. (3), $\frac{d^2P}{dV^2} = 2RT_0.V^{-3} > 0$

$\therefore$ P is minimum at $V = \sqrt{\frac{T_0}{\alpha}}$

$\therefore$ from eq. (2), $P_{min} = R\left(T_0.\sqrt{\frac{\alpha}{T_0}} + \sqrt{\alpha T_0}\right)$

$\Rightarrow P_{min} = R\left(\sqrt{\alpha T_0} + \sqrt{\alpha T_0}\right)$

$\Rightarrow P_{min} = 2R\sqrt{\alpha T_0}$

5. (A)

It E be the emf of cell C, when switch S is open, in the balanced condition.

$E = kl_1$ ....(1), $(l_1 = 76.3\ cm)$

where K is potential gradient of the potentiometer wire when switch S is closed, in the balanced condition,

$E - iR = k\,l_2$ $(l_2 = 60.0$ cm)

where l is the current in the external circuit,

$\Rightarrow e - \left(\frac{e}{R+r}\right)r. = kl_2$

(where r is internal resistance of cell C)

$\Rightarrow \frac{ER}{R+r} = kl_2$ .....(2)

Dividing eq. (2) by (1), $\frac{R}{R+r} = \frac{l_2}{l_1}$

$\Rightarrow \quad \frac{R+r}{R} = \frac{l_1}{l_2}$

$\Rightarrow \quad \frac{r}{R} = \frac{l_1}{l_2} - 1$

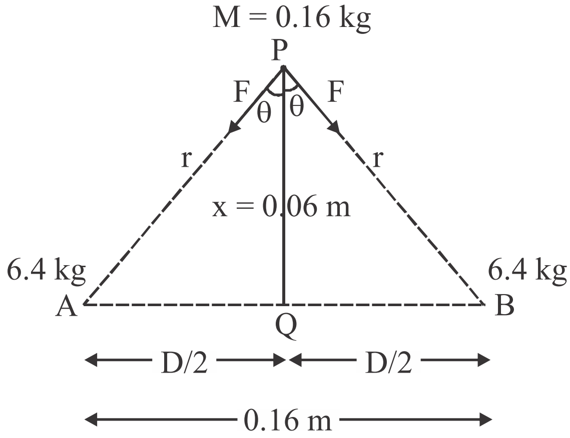
$\Rightarrow \quad r = R\left(\frac{l_1}{l_2} - 1\right) = 4\,\Omega \times \left(\frac{76.3}{60.0} - 1\right)$

$\Rightarrow \quad r \approx 1\,\Omega$

6. (C)

Let $M_A = M_B = M'$ $(=6.4 \text{ kg})$

Gravitational force between M and M_A (or between M and M_B) is



$$F = \frac{GMM'}{r^2}$$

By symmetry, the net force is along PQ.

$\therefore$ net force is $F_{net} = 2F\cos\theta = \frac{2GMM'}{r^2}.\frac{x}{r}$

$\therefore$ acceleration of mass M at P, is

$$a = \frac{F_{net}}{M} = \frac{2GM'x}{r^3} = \frac{2\times 6.67\times 10^{-11}\times 6.4\times 0.06}{\left\{(0.06)^2 + \left(\frac{0.16}{2}\right)^2\right\}^{\frac{3}{2}}}$$

$$\Rightarrow a = \frac{5.122}{0.001}\times 10^{-11} ms^{-2} = 5.122\times 10^{-8} ms^{-2}$$

Now, decrease in gravitational P.E. of the system as the mass M moves from P to Q, is equal to increase in K.E.

$$\therefore \frac{1}{2}MV^2 = 2\times\left(\frac{-GMM'}{r}\right) - 2\times\left(\frac{-GMM'}{D/2}\right)$$

$$\Rightarrow \frac{1}{2}MV^2 = 2GMM'\left(\frac{2}{D} - \frac{1}{r}\right)$$

$$\Rightarrow V^2 = 4GM'\left(\frac{2}{D} - \frac{1}{r}\right) = 4\times 6.67\times 10^{-11}\times 6.4\times\left(\frac{2}{0.16}\times\frac{1}{0.1}\right)$$

$$\Rightarrow V^2 = 170.752\times 10^{-11}\times 2.5 = 42.688\times 10^{-10}$$

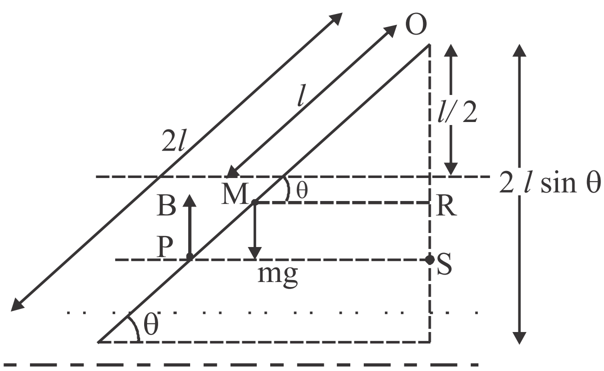
$$\Rightarrow V \approx 6.5\times 10^{-5} m/s$$

7. (D)

Let Q be the angle of inclination of the rod with the horizontal. Then length of the rod inside water is then

$$lw = 2l - \frac{l}{2}.\cos\sec\theta$$

So, the buoyant force will act on the mid point P of the part of the rod inside water.



$$\therefore OP = 2l - \frac{1}{2}l_w = 2l - \left(\frac{2l - \frac{l}{2}\cos ec\theta}{2}\right)$$

or,, $OP = \ell + \frac{l}{4}\cos esc\,\theta$

Now, if M be the mid point of rod, the weight of the rod acts at M where, $OM = \ell$.

For equilibrium of the rod, the torque of buoyant force about point O is balanced torque of weight.

$$\therefore B(OP\cos\theta) = mg\left(OM\cos\theta\right)$$

$$\Rightarrow B.\left(OP\right) = mg.\left(OM\right)$$

$$\Rightarrow \left[\rho_w.\left\{\frac{l\ell}{2\ell}.V\right\}g\right]\left(OP\right) = \left(\rho_r.V\right)g\left(OM\right)$$

where, ρ_w and ρ_r are densities of water and rod respectively and V is volume of the rod.

$$\Rightarrow \frac{\ell_w}{2\ell}.\left(OP\right) = \left(\frac{\rho_r}{\rho_w}\right).\left(OM\right)$$

$$\Rightarrow \left(1-\frac{1}{4}\cos ec\theta\right)\left(OP\right) = 0.75\,\ell$$

$$\left\{\because \ell_w = 2\ell - \frac{\ell}{2}\cos ec\,\theta\right\}$$

$$\Rightarrow \left(1-\frac{1}{4}\cos ec\,\theta\right)\left(l+\frac{\ell}{4}\cos ec\theta\right) = 0.75\,\ell$$

$$\Rightarrow \ell^2 - \left(\frac{1}{4}\cos ec\theta\right)^2 = 0.75$$

$$\Rightarrow \frac{1}{16}\cos ec^2\theta = 0.25$$

$$\Rightarrow \cos ec^2\theta = 4 \Rightarrow \cos ec\theta = 2 \Rightarrow \sin\theta = \frac{1}{2}$$

$$\Rightarrow \theta = 30^0$$

8. (D)

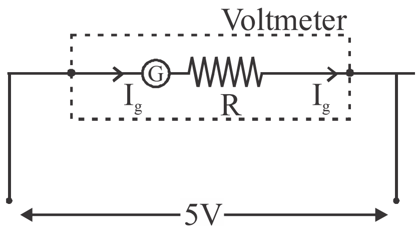
Let G be the resistance of galvanometer.

$R = 975\,\Omega$

For maximum voltage 5V, there will be full scale deflection in galvanometer.

So, at this times current through galvenometer is $I_g = 0.005$ A

$$\therefore\; V_g + V_R = V \qquad (V = 5V)$$



$$\Rightarrow\; I_g.G + I_g - R = V$$

$$\Rightarrow\; I_g.G = V - I_gR$$

$$\Rightarrow\; G = \frac{V}{I_g} - R = \frac{5V}{0.005A} - 975\,\Omega = 25\,\Omega$$

9. (D)

The magnetic moment of the current carrying coil, is $M = NIA = NI\pi R^2$

$\therefore$ Torqu experienced by the sphere - coil system, is

$\vec{\tau} = \vec{M} \times \vec{B}$

$\therefore \tau = MB$ $(\because \vec{M} \perp \vec{B}, \vec{M}$ being vertical)

$\therefore$ angular acceleration,

$$\alpha = \frac{\tau}{M.O.I} = \frac{MB}{\frac{2}{5}MR^2} = \frac{5MB}{2mR^2}$$

$$\therefore \alpha = \frac{5B}{2mR^2} \times NI\pi R^2 \qquad (\because M = NI\pi R^2)$$

$$\Rightarrow \alpha = \frac{5N\pi IB}{2m}$$

10. (C)

The object is kept at 2F point of the convex lens, so image formed by convex lens will also be at 2F (on the other side of lens) and of same size. This image acts as virtual object for concave lens.

The distance of this virtual object from concave lens is thus 40 cm - 8 cm = 32 cm

$\therefore$ for concave lens,

$u = +32\ cm, f = -10\ cm. h_1 = -1cm$ (height of inverted virtual object)

$\therefore$ using $\frac{1}{V} - \frac{1}{u} = \frac{1}{f}$, we get

$$\frac{1}{V} = \frac{1}{u} + \frac{1}{f} = \frac{1}{32} - \frac{1}{10} = \frac{-11}{160}$$

$$\Rightarrow v = -\frac{160}{11} cm \approx -14.5\ cm$$

$\therefore$ the image will be formed at a distance of 14.5 cm on the left side of concave lens.

magnification produced by this lens, is

$$m = \frac{v}{u} = \frac{-14.5\ cm}{32\ cm} \approx -0.45$$

$$\Rightarrow \frac{h_2}{h_1} = -0.45 \qquad \Rightarrow \qquad h_2 = -0.45 \times h_1$$

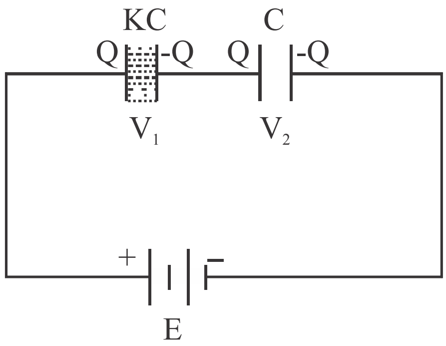
$$\Rightarrow h_2 = -0.45 \times (-1\ cm) = 0.45\ cm$$

11. (B)

The velocity acquired by the falling part of the chain after a length x of the chain has falllen, is

$$v = \sqrt{gx} \qquad (v^2 = gx) \qquad .....(1)$$

considering a small element of length dx which falls in a time dt, the momentum of this part just before striking the weighing machine is $(dm).v = \left(\frac{Mdx}{\ell}\right)v$



where $dm = \frac{Mdx}{\ell}$ is mass of this element of chain. This element comes to rest after striking the weighing machine.

$\therefore$ loss in momentum $= \left(\frac{Mdx}{l}\right)v$

which is the momentum imparted to weighing machine in time dt by the chain.

$\therefore$ force exerted by this element is the momentum imparted per unit time $= \frac{\left(\frac{Mdx}{\ell}\right)v}{dt} = \frac{Mv}{\ell}.\frac{dx}{dt}$

$$= \frac{Mv}{\ell} \times v = \frac{Mv^2}{\ell} = \frac{M}{\ell} \times = gx = \frac{Mgx}{\ell}$$

Also, the force exerted by the part of the chain resing on weighing machine = weight of that part

$$= \left(\frac{Mx}{\ell}\right).g = \frac{Mgx}{\ell}$$

$\therefore$ Total force exerted by the chain on weighing machine $= \frac{Mgx}{\ell} + \frac{Mgx}{\ell} = \frac{2Mgx}{\ell}$

12. (B)

Initial total energy of the capacitors $= \frac{1}{2}CV^2 + \frac{1}{2}CV^2 = CV^2$

As both the capacitors are equal, each will have a voltage drop $V = \frac{E}{2}$

$\therefore$ Initial total energy $= \frac{1}{4}CE^2$ ......(i)

When one capacitoris filled with dielectric of dielectric constant k, its capacitance becomes KC

$\therefore$ if V_1 and V_2 be the p.d. across the two capacitors respectively,

$$\frac{V_1}{V_2} = \frac{C_2}{C_1} = \frac{C}{KC} = \frac{1}{K}$$

$$\Rightarrow V_2 = KV_1$$

$$\because\ V_1 + V_2 = E \Rightarrow V_1 + KV_1 = E$$

$$\Rightarrow (K+1)V_1 = E \qquad \Rightarrow V_1 = \frac{E}{K+1}$$

$\therefore \quad V_2 = \dfrac{KE}{K+1}$

$\therefore$ total energy of capacitors in final state =

$$\frac{1}{2}(KC)V_1^2 + \frac{1}{2}CV_2^2 = \frac{1}{2}(KC).\left(\frac{E}{K+1}\right)^2 + \frac{1}{2}C.\left(\frac{KE}{K+1}\right)^2$$

$$= \frac{1}{2}CE^2\left(\frac{K}{(K+1)^2} + \frac{K^2}{(K+1)^2}\right)$$

$$= \frac{1}{2}CE^2 \times \frac{K(K+1)}{(K+1)^2} = \frac{1}{2}CE^2.\left(\frac{K}{K+1}\right)$$

$\therefore$ Final total energy $= \dfrac{1}{2}\left(\dfrac{K}{K+1}\right)CE^2$ ......(2)

$\therefore$ from eq. (1) and (2) the ratio of final total energy and initial total energy is

$$\frac{U_2}{U_1} = \frac{\frac{1}{2}\left(\frac{K}{K+1}\right).CE^2}{\frac{1}{4}CE^2} = \frac{2K}{K+1} \Rightarrow U_2 = \left(\frac{2K}{K+1}\right)U_1$$

$$\therefore x = \frac{2K}{K+1}$$

13. (C)

Since cells are connected in series quantity of current flowering through then is the same

By applying Faraday's second law of electrolysis,

$$\frac{m_1}{m_2} = \frac{E_1}{E_2}$$

$$\frac{m_{Au}}{m_{Cu}} = \frac{E_{Au}}{E_{Cu}}$$

$$\frac{10}{M_{Cu}} = \frac{(197/3)}{(63.5/2)}$$

$$\therefore M_{Cu} = 4.835g$$

By applying Faraday's First law,

$$\frac{M_{Cu}}{E_{Ca}} = \frac{i \times t}{96500}$$

$$\frac{4.835}{(63.5/2)} = \frac{i \times 4 \times 60 \times 60}{96500}$$

$$\therefore i = 1.021A$$

14. (B)

(A) $\left.\begin{matrix} NF_3 \\ H_3O^{\oplus} \end{matrix}\right\}$ Steric no – 4, with one L.P (Pyramidal)

(B) $NO_2^{\oplus} - sp -$ Luniar

$SO_2 - sp^2 -$ Bent

(C) $\left.\begin{matrix} PCl_5 \\ CH_3 \end{matrix}\right\}$ Steric no - 4,with on L.P

15. (C)

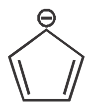
$P_b(NO_3)_2 + NaCl \longrightarrow PbCl_2 + NaNO_3$

$Pb(NO_3)_2$ —dil HCl, Δ→ Reddish brown (gas) NO_2

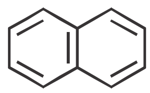
$PbCl_2$ white ppt (Q) —H_2S→ Pbs black ppt

$PbCl_2$ white ppt (Q) —hot water NaI→ PbI_2 Yellow ppt

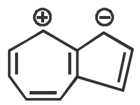
16. (A)



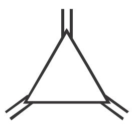
— Aroamatic (4n + 2) n = 1



— Aroamatic (4n + 2) n = 2



— Aroamatic (4n + 2) n = 2



— Non - Aromatic

17. (D)

18. (A)

$$P_{ideal} = \frac{nRT}{V} = \frac{2.27 \times 0.082 \times 313}{5} = 11.652\,atm$$

$$P_{ideal} > P_{real}$$

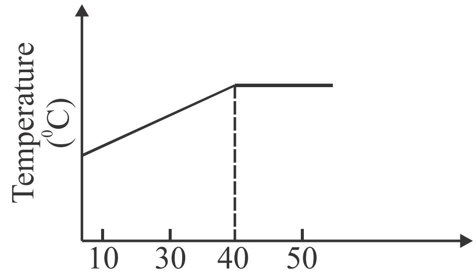
19. (D)

$\left(N_1V_1\right)_{Acid} = \left(N_2V_2\right)_{Base}$

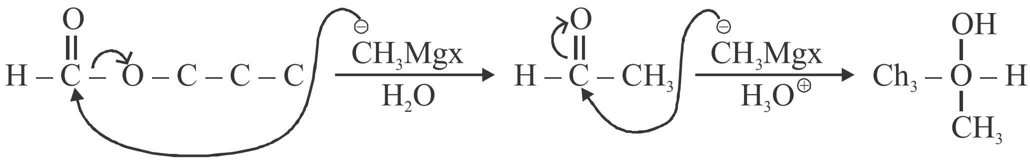
$\left(n_1 \times M_1 \times V_1\right)_{Acid} = \left(n_2 \times M_2 \times V_2\right)_{Base}$

$1 \times 0.2 \times 100 = 2 \times 0.25 \times V_2$

One Neutralisation is complete, 'V_2' of H_2SO_4 will remain constant.

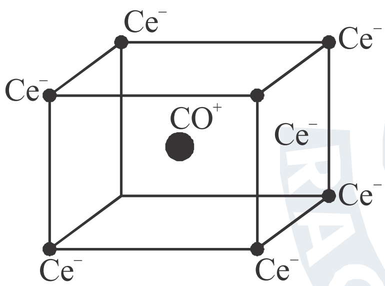


Vol. of $H_2SO_4\left(cm^3\right)$



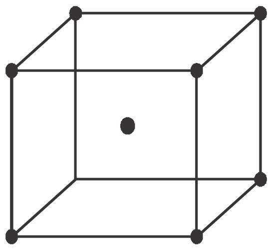
(X) (Y)

20. (1) $CsCl$ crystal structure along with coordination is shown below. The Co^+ ion is surrounded by $8Ce^-$ ions at the corners of unit cell.



$\therefore$option$(1)$is correct

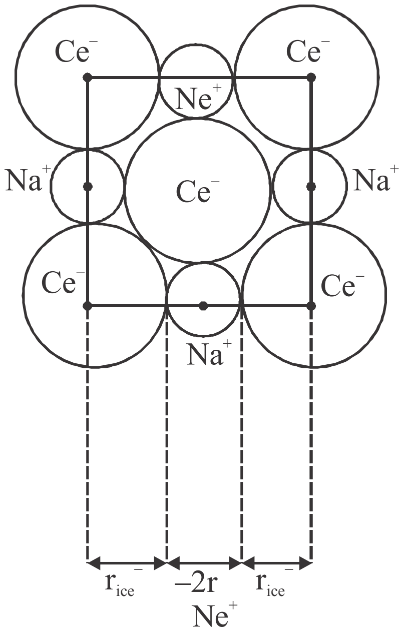
(2) BCC structure of metals is shown below. The metal atom at the centre of the body is surrounded by 8 non-metal atoms that are at teh corners of the cube



(3) Every unit cell shares some of its atom / ions with other unit cell

$\therefore$option$(2)$is correct

(4) Unit cell of $NaCl$ is shown below. The Cl^- ions are arranged at the corners as well as at the centers of the faces, while Na^+ ions occupy position at the body centre. The Na^+ & Cl^- ions are touching each other along the edge of the unit cell.



$\therefore$ Edge length = $\left(2r_{Ne^+}\right)+\left(2r_{cl^-}\right)$

$=\left(2\times 95\right)+\left(2\times 181\right)$

$=552\,pm$

option (4) is correct

21. molar weight of urea $\left(NH_2CONH_2\right)=60g$

6% (w/v) of (Urea means 69 g urea in 100ml solution)

$\therefore n_{urea}=\frac{6}{60}=0.1 moles$

Molarity (urea) $=\frac{0.1}{\left(100/1000\right)}=1M$

(I) Molar wt. of Glucose = 180g

18% (w/v) of Glucose means 18g Glucose in 100ml solution

$\therefore n_{\text{Glucose}}=\frac{18}{180}=0.1 moles$

$\therefore$ Molarity(Glucose) $=\frac{0.1}{\left(100/1000\right)}=1M$

Option (I) is correct

(II) 0.5 M $NaCl$ is not correct

(III) 1M CH_3COOH is correct

(IV) Molar weight of sucrose $\left(C_{12}H_{22}O_{11}\right)=342g$

6% (w/v) of sucrose means 6g sucrose in 100ml solution

$\therefore^{n}$ Sucrose $=\frac{6}{342}=0.017\,moles$

$$\text{Molarity} = (\text{Sucrose}) = \frac{0.017}{0.1} = 0.17M$$

Option (IV) is incorrect

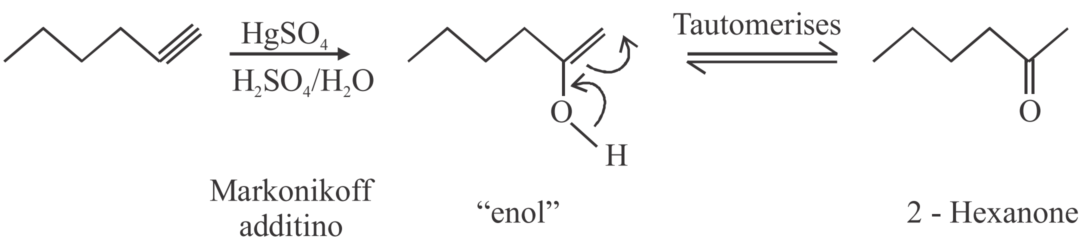
Ans – None of the option are correct

22. (A)

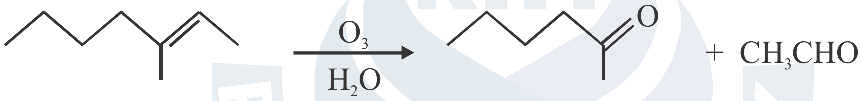
In this case nitrogen with pyramidal shape undergoes rapid inversion of shape, so it can never be resolved.

23. (A)

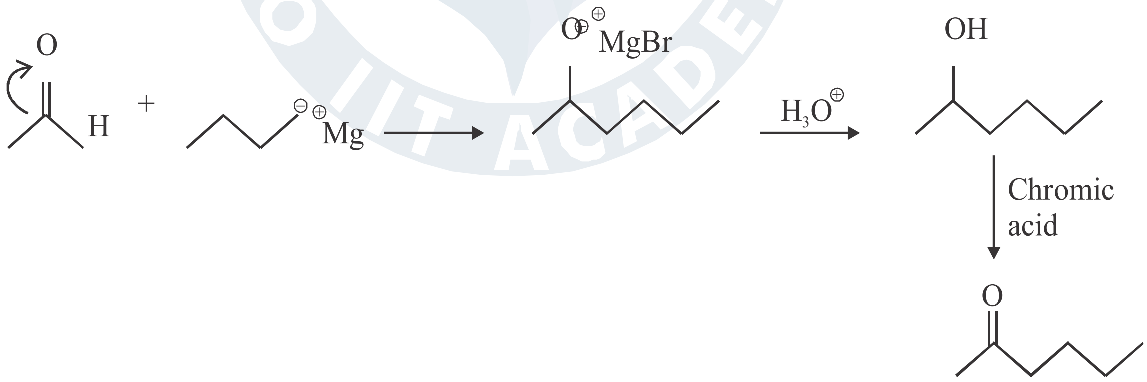
I



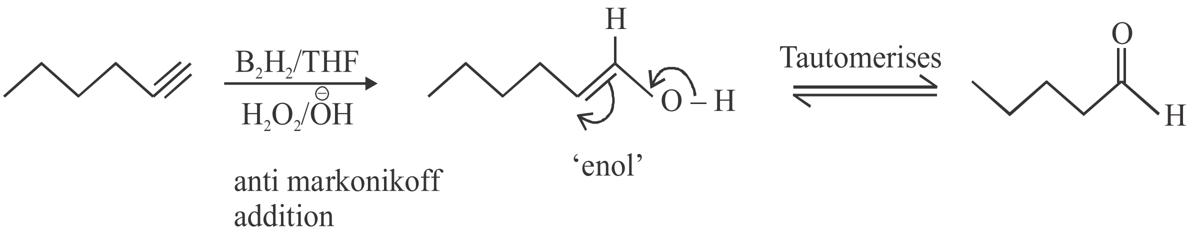
II



III



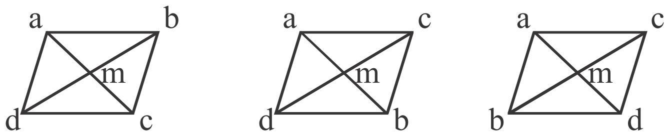
IV



24. (A)

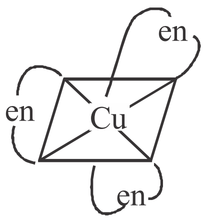
I $\left[Cu\left(NH_3\right)_4\right]^{2+}$ - sp^2d hybridisation and paramagnetic.

II $\left[Pt\left(NH_3\right)\left(Cl\right)\left(Py\right)\left(Br\right)\right]$ - it is M_{abcd} type

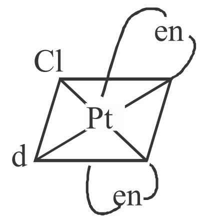


3 - qgeometrical isomer.

III



Optically active



Optically active

IV CFSe depends on magnitude of change and position of element in periodic table.

CFS α magnitude of change

CF increases down the group.

25. (B)

$4P \rightarrow 3Q + R$

$\frac{-r_P}{4} = \frac{r_Q}{3} = \frac{r_R}{1}$

$\frac{K_1[P]^4}{4} = \frac{K_2[P]^4}{3} = \frac{K_3[P]^4}{1}$

$\therefore 3K_1 = 4K_2 = 12K_3$

26. (B)

$O'S_3 = O'S_4 = \frac{z}{2}$

path difference between the waves from $S_1\, and\, S_2$ meeting at $S_3\left(or\, S_4\right), \Delta x = \frac{\left(\frac{z}{2}\right).d}{D} = \frac{zd}{2D}$

phase difference, $\phi = \frac{2\pi}{\lambda} \times \Delta x = \frac{2\pi}{\lambda} \times \left(\frac{zd}{2D}\right)$

when $z = \frac{\lambda D}{2d}, \phi = \frac{2\pi}{\lambda} \times \frac{\lambda}{4} = \frac{\pi}{2}$

if I_m be the maximum intensity the intensity of light at S_3.

(or S_4) will be $I = I_m.\cos^2\frac{\phi}{2} = I_m.\cos^2\frac{\pi}{4} = \frac{I_m}{2}$

Intensity at O in this case is

$I_o = 4I = 2I_m$ ____ (i)

Now,when $z = \frac{2\lambda D}{d}, \phi = \frac{2\pi}{\lambda}\times\left(\frac{zd}{2D}\right) = \frac{2\pi}{\lambda}\times\lambda$

or, 2π

the intensity of light at S_3 (or S_4) will be

$I = I_m\cos^2\frac{\phi}{2} = I_m\cos^2\pi = I_m$

Intensity at O in this case is 4I = $4I_m$

$= 2I_0$ {using equation (i)}

$\therefore E_x = 1$ and $E_y = 0 \therefore \tan\theta = \frac{E_y}{E_x} = 0 \Rightarrow \theta = 0^0$

For figures given in (R), we have

$\frac{\delta v}{\delta x} = \tan 45^0 = 1$ and $\frac{\delta V}{\delta Y} = \tan 120^o = -\sqrt{3}$

$\therefore E_x = -1$ and $E_y = \sqrt{3}$

$\therefore \tan\theta = \frac{E_y}{E_x} = \frac{\sqrt{3}}{1/\sqrt{3}} = 3$

$\Rightarrow \theta = \tan^{-1}(3)$

27. (C)

In this case intensity of light at O' will be maximum i.e. I_m.

For S_3, the path difference $\Delta x = \frac{(O'5_3)d}{D} = \frac{\left(\frac{\lambda D}{4d}\right).d}{D}$

or, $\Delta x = \frac{\lambda}{4}$

phase difference $= \phi = \frac{2\pi}{\lambda}.\Delta x = \frac{\pi}{2}$

Intensity of light at S_3, will be $I = I_m\cos^2\frac{\phi}{2} = I_m\cos^2\frac{\pi}{4}$ or $I = \frac{I_m}{2}$

Amplitude ratio for slits S_3 and O' is

$r = \frac{A_1}{A_2} = \sqrt{\frac{I_1}{I_2}} = \sqrt{\frac{Ir_2}{I_m}} = \frac{1}{\sqrt{2}}$

ratio of maximum intensity to minimum intensity on the screen

$$\frac{I_{max}}{I_{min}} = \left(\frac{r+1}{r-1}\right)^2 = \left(\frac{\frac{1}{\sqrt{2}}+1}{\frac{1}{\sqrt{2}}-1}\right)^2 \approx 34$$

28. (*)

From the figures given in (P), we get

$$\frac{\delta v}{\delta x} = \tan 45^o \text{ (slope of graph)} = 1$$

also, $\frac{\delta V}{\delta Y} = \tan 150^o = -1/3 \therefore E_Y = \frac{-\delta V}{\delta Y} = \frac{1}{\sqrt{3}}$

if θ is the angle made by $\vec{E}$ with positive direction of $x-$axis, $\tan\theta = \frac{E_Y}{E_X} = \frac{-1}{\sqrt{3}} \Rightarrow \theta = 150^o$

for the figures in (Q), we get

$$\frac{\delta V}{\delta X} = \tan 135^0 = -1 \text{ and } \frac{\delta V}{\delta Y} = 0$$

29. (B)

I - As it is followoing carbocation mechanism it is E_1 - elimination.

II - Rate determining step is formation of carbocation.

III - As it is E1 = order of reactivity should be $3^0 > 2^0 > 1^0$.

IV - It follows saytself rule.

30. (C)

$$4H_3PO_3 \xrightarrow{\Delta} 3H_3PO_4 + PH_{3(g)}$$

$$3AgNO_3 + PH_3 \longrightarrow \underset{\text{Black ppt}}{Ag_3P} \downarrow 3HNO_3$$

31. (C)

$(x-\alpha)^2 + (y-\beta)^2 = r^2$ (M) {where α, β are orbitarary constant}

$x^2 + y^2 - 2\times x - 2\beta y + \alpha^2 + \beta^2 - r^2 = 0$ .....(1)

$\Rightarrow \quad 2x + 2yy' - 2\alpha - 2\beta y' = 0$ .....(2)

$\Rightarrow \quad 2 + 2(y')^2 - 2yy'' - 2\beta y'' = 0$ ......(3)

from eq. (3) $\Rightarrow$ $\boxed{\beta = \frac{(y')^2 + (y)(y'') + 1}{(y'')}}$

from eq. (2) $\Rightarrow$ $\boxed{\alpha = x + yy' - \frac{\left((y')^3 + yy'y'' + y\right)}{y''}}$

Using values of α and β in equation (M)

$$\left[x-\left(x+yy'-\frac{\left((y')^3+yy'y''+y'\right)}{y''}\right)\right]^2+\left[y-\left(\frac{(y')^2+yy''+1}{y''}\right)\right]^2=r^2$$

$$\Rightarrow \quad \left(\frac{(y')^3+yy'y''+y'}{y''}-yy'\right)^2+\left(\frac{yy''-(y')^2+yy''+1}{y''}\right)^2=r^2$$

$$\Rightarrow \quad \left(\frac{(y')^3+y'}{y''}\right)^2+\left(\frac{(y')^2+1}{y''}\right)^2=r^2$$

$$\Rightarrow \quad \left((y')^3+(y')\right)^2+\left((y')^2+1\right)^2=r^2(y'')^2$$

$$\Rightarrow \quad (y')^2\left((y')^2+1\right)^2+\left((y')^2+1\right)^2=r^1(y'')^2$$

$$\Rightarrow \quad \left((y')^2+1\right)^2\left[(y')^2+1\right]=r^2(y'')^2$$

$$\Rightarrow \quad \boxed{\left[(y')^2+1\right]^3=r^2(y'')^2}$$

32. (A)

$${}^{n+2}C_8 = {}^{n-2}P_4 = 57:16$$

$$\frac{(n+2)!}{8!(n-6)!}\times\frac{(n-6)!}{(n-2)!}=\frac{57}{16}$$

$$\frac{(n+2)(n+1)(n)(n-1)(n-2)!}{8!(n-2)!}=\frac{57}{16}$$

$$(n+2)(n+1)(n)(n-1)=\frac{57\times8\times7\times6\times120}{16}$$

$$=3\times19\times7\times3\times6\times20$$

$$=21\times20\times19\times18$$

On comparing n = 19

33. (B)

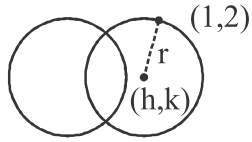
$\because$ elements of matrix are integers

$\because$ co - factor matrix and transpose of co- factor matrix will also have integral entries.

Now : Determinant is $\pm$ 1

Inverse matrix exist and all entries are integers.

34. (C)



Let (h, k) be the center then :

$$(x-h)^2+(y-k)^2=(1-h)^2+(2-k)^2$$

$$\Rightarrow \boxed{x^2+y^2-2hx-2ky+(2h+4k-5)=0} \qquad \text{- circle (1)}$$

Also :

$$\boxed{x^2+y^2-4=0} \qquad \text{- circle (2)}$$

from circle (1) $\begin{cases} g_1=-h; \quad f_1=-k \\ c_1=2h+4k-5 \end{cases}$

from circle (2) $\begin{cases} g_2=0; \quad f_2=0 \\ c_2=-4 \end{cases}$

if circle (1) and circle (2) are orthogonal then ;

$$2g_1g_2+2f_1f_2=c_1+c_2$$

$$2(\bullet h)(0)+2(-k)(0)=(2h+4k-5)+(-4)$$

$$\Rightarrow \quad 2h+4k-9=0$$

$\therefore$ locus will be $\boxed{2x+4y-9=0}$

35. (D)

$$f(x)=(ax^2+b)^2$$

$fog(x)=g\ of(x)=x$ for $g(n)=f^{-1}(x)$

to get $f^{-1}(n)$; represent 'x' in terms of 'y'

$$y=(ax^2+b)^3$$

$$y^{1/3}=ax^2+b$$

$$x^2=\left(\frac{y^{1/3}-b}{a}\right) \qquad x=\left(\frac{y^{1/3}-b}{a}\right)^{1/2}$$

Now; seperate 'x' with 'y' with 'n'

$$\therefore \quad y=\left(\frac{x^{1/3}-b}{a}\right)^{1/2}=g(x)$$

36. (A)

$$A.P.\begin{cases}\text{Common difference : D}\\ \text{First Term : A}\end{cases}$$

$$\Rightarrow \begin{cases}T_P = A+(p-1)D = a & ....(1)\\ T_q = A+(q-1)D = b & ....(2)\\ T_r = A+(r-1)D = c & ....(3.)\end{cases}$$

(1) - (2) $\Rightarrow (p-q)D = a-b$

(2) - (3) $\Rightarrow (q-r)D = (b-c)$

on division $\boxed{\left(\frac{p-q}{q-r}\right) = \left(\frac{a-b}{b-c}\right)}$ ..... (M)

$$G.P.\begin{cases}\text{First term : B}\\ \text{common ratio : R}\end{cases}$$

$$\Rightarrow G.P.\begin{cases}Tp = B(R)^{P-1} = a & (4)\\ T_q = B(R)^{q-1} = b & (5)\\ T_r = B(R)^{r-1} = c & (6)\end{cases}$$

(4) - (5) $\Rightarrow (R)^{p-q} = a/b$

(5) - (6) $\Rightarrow (R)^{q-r} = b/c$

take log and divide

$$\frac{p-q}{q-r} = \frac{\log(a/b)}{\log(b/c)} \quad .... (N)$$

from eq. (M) and (N)

$$\left(\frac{a-b}{b-c}\right) = \frac{\log(a/b)}{\log(b/c)}$$

$$\Rightarrow (a-b)\log\left(\frac{b}{c}\right) = (b-c)\log\left(\frac{a}{b}\right)$$

$$\Rightarrow \left(\frac{b}{c}\right)^{a-b} = \left(\frac{a}{b}\right)^{b-c}$$

$$\Rightarrow (a)^{b-c}(b)^{c-a}(c)^{a-b} = 1$$

37. (A)

Shortest distance is along the normal from (0, 0) to the curve.

$\because \quad y = \dfrac{e^x + e^{-x}}{2}$

$\Rightarrow \quad y' = \dfrac{e^x - e^{-x}}{2}$

$\begin{cases} \text{i.e. at } x = 0 \\ y' = 0 \end{cases}$

$\Rightarrow$ y - axis is normal to the curve.

$\therefore$ intersection point of curve of normal from (0, 0) is (0, 1).

$\therefore$ shortest distance = 1 unit.

38. (D)

$$f(x) = \begin{cases} \dfrac{k\cos x}{\pi - 2x}; & if \ x \neq \pi/2 \\ 3 & ;if \ x = \pi/2 \end{cases}$$

for continuity LHL = RHL = 3

$\Rightarrow \quad \lim\limits_{x \to 0} \left(\dfrac{k\cos x}{\pi - 2x} \right) = 3 \qquad \Rightarrow \quad \lim\limits_{x \to 0} \dfrac{k\cos(\pi/3 + h)}{\pi - 2\left(\dfrac{\pi}{2} + h\right)} = 3$

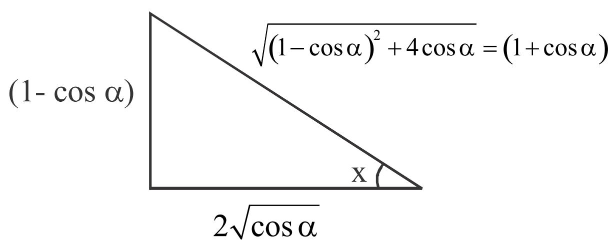
$\Rightarrow \quad \lim\limits_{x \to 0} \dfrac{k.(-\sin h)}{-2h} = 3 \qquad = \dfrac{k}{2} = 3 \quad \Rightarrow \quad \boxed{k = 6}$

39. (A)

$\cot^{-1}\left(\sqrt{\cos\alpha}\right) - \tan^{-1}\left(\sqrt{\cos\alpha}\right) = x$

$\Rightarrow \quad \tan^{-1}\left(\dfrac{1}{\sqrt{\cos\alpha}}\right) - \tan^{-1}\left(\sqrt{\cos\alpha}\right) = x$

taking 'tan' both sides



$\Rightarrow \quad \dfrac{\left(\dfrac{1}{\sqrt{\cos\alpha}} - \sqrt{\cos\alpha}\right)}{1 + \left(\dfrac{1}{\sqrt{\cos\alpha}}\right)\left(\sqrt{\cos\alpha}\right)} = \tan x$

$\Rightarrow \quad \dfrac{1 - \cos\alpha}{2\sqrt{\cos\alpha}} = \tan x$

$\therefore \quad \sin x = \left(\dfrac{1 - \cos\alpha}{1 + \cos\alpha}\right)$

$= \dfrac{2\sin^2\alpha/2}{2\cos^2\alpha/2} \qquad = \tan^2\alpha/2$

40. (B)

$$\int_{-3\pi/2}^{-\pi/2}\left((x+\pi)^3+\cos^2 x\right)dx$$

Let ; $x+\pi = t \Rightarrow dx = dt$

$$\int_{-\pi/2}^{\pi/2}\left(t^3+\cos^2 t\right)dt \qquad = \int_{-\pi/2}^{\pi/2} t^3 dt + \int_{-\pi/2}^{\pi/2}\cos^2 t \; dt$$

t^3 is odd $\therefore$ integral $= 0$

$$= 2\int_0^{\pi/2}\sin^2 + dt \qquad = \int_0^{\pi/2}\left(1-\cos^2 t\right)dt$$

$$= \left(t-\frac{\sin 2t}{2}\right)_0^{\pi/2} \qquad = \frac{\pi}{2}$$

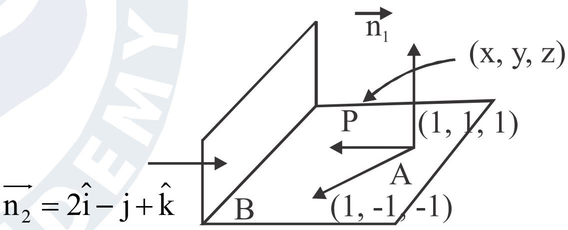
41. (B)

$\vec{n_1} = \overrightarrow{AB}\times\overrightarrow{AP}$

$$= \begin{vmatrix} i & j & k \\ 0 & -2 & -2 \\ x-1 & y-1 & z-1 \end{vmatrix}$$

$$= \left(-2(z-1)+2(y-1)\right)\hat{i} - \left(2(x-1)\right)\hat{j} + \left(2(x-1)\right)\hat{k}$$

$\because$ $\vec{n_1}$ and $\vec{n_2}$ are perpendicular.

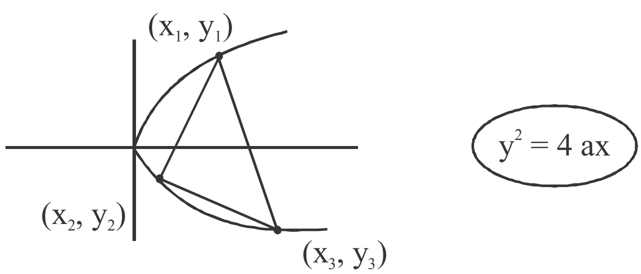


$\Rightarrow \vec{n_1}\cdot\vec{n_2} = 0 \Rightarrow -4z+4+4y-4+2x-2+2x-z = 0$

$\Rightarrow 4x+4y-4z-4 = 0 \Rightarrow \boxed{x+y-z=0}$

42. (D)

$$Area = \left|\frac{1}{2}\begin{vmatrix} x_1 & x_2 & x_3 \\ y_1 & y_2 & y_3 \\ z_1 & z_2 & z_3 \end{vmatrix}\right|$$



$$= \left|\frac{1}{2}\begin{vmatrix} y_1 2/4a & x_2 2/4a & x_3 2/4a \\ y_1 & y_2 & y_3 \\ 1 & 1 & 1 \end{vmatrix}\right|$$

$$= \left|\frac{1}{8a}\begin{vmatrix} y_1^2 & y_2^2 & y_3^2 \\ y_1 & y_2 & y_3 \\ 1 & 1 & 1 \end{vmatrix}\right| \qquad = \left|\frac{1}{8a}\begin{vmatrix} y_1^2-y_3^2 & y_2^2-y_3^2 & y_3^2 \\ y_1-y_3 & y_2-y_3 & y_3 \\ 0 & 0 & 1 \end{vmatrix}\right|$$

$$=\left|\frac{(y_1-y_3)(y_2-y_3)}{8a}\right|\begin{vmatrix}(y_1+y_3)&(y_2+y_3)&y_3\\1&1&y_3\\0&0&1\end{vmatrix} \quad =\left|\frac{(y_1-y_3)(y_2-y_3)}{8z}\times\left((y_1+y_3)-(y_2+y_3)\right)\right|$$

$$=\frac{1}{8a}\left|(y_1-y_2)(y_1-y_3)(y_3-y_1)\right|$$

$\therefore k=1/8$

43. (C)

$\boxed{a*b-=|a-b|}$

Now : (i) $b*a=|b-a|=|a-b|$

$\Rightarrow$ * is cummutative

(ii) $(a*b)*c=|a-b|*c$

$=|a-b|-c|$

$\Rightarrow *$ is not associative.

a . b = a

Now : (iii) $\left.\begin{matrix}a\,.\,b=a\\ \&\ b\,.\,a=b\end{matrix}\right\}\because a\,.\,b\neq b\,.\,a\Rightarrow$ '0'is not commutative

(iv) $(a.b).C==a.c=a$

and $a.(b\,.\,c)=a\,.\,b=a$

$\Rightarrow$ associative.

44. (A) $=(1+x^2)y=2-x$

at x - axis $y=0$

$\Rightarrow (1+x^2)(0)=2-x$

$\Rightarrow \boxed{x=2}$

$\therefore$ tangent drawn at (2, 0)

Now : for slope at (2, 0)

$(1+x^2)y=2-x$

$\Rightarrow (2x)y+(1+x^2)\frac{dy}{dx}=-1$

$\Rightarrow \frac{dy}{dx}=\frac{-(1+2xy)}{(1+x^2)}$

$\Rightarrow \left(\dfrac{dy}{dx}\right)_{(2,0)} = \dfrac{-(1+0)}{1+4} = \dfrac{-1}{5}$

eq. of tangent

$(y-0) = \dfrac{-1}{5}(x-2)$

$5y = -x+2$

$x+5y-2=0$

45. (C)

$\vec{a}+2\vec{b}+3\vec{c}$

$0.\vec{a}+\lambda\vec{b}+3\vec{c}$

$0.\vec{a}+0.\vec{b}+(2\lambda-1)\vec{c}$

for coplamarity

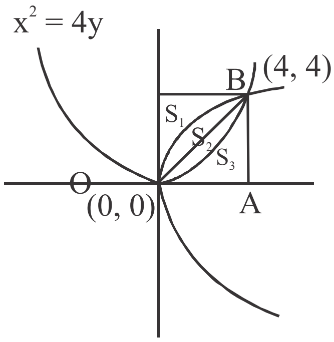
$$\begin{vmatrix} 1 & 2 & 3 \\ & \lambda & 3 \\ 0 & 0 & (2\lambda-1) \end{vmatrix} = 0$$

$\Rightarrow (2\lambda-1)(\lambda-0)=0$

$\Rightarrow \lambda = 0$ and $1/2$

46. (B)

$5_2 = 2\left[\dfrac{1}{2}\times4\times4-\int_0^4\dfrac{x^2}{4}dx\right]$



$= 2\left[8-\left(\dfrac{x^3}{3}\right)_0^4\times\dfrac{1}{4}\right]$

$= 2\left[8-\dfrac{16}{3}\right]$

$= \dfrac{16}{3}$

Also, $5_1 = \dfrac{16}{3}$ and $5_3 = \dfrac{16}{3}$

47. (A)

Probability of both aces $P(A_2) = \frac{4}{52} \times \frac{3}{51}$

Probability of + suit $= P(T) = \frac{60}{100} = \frac{3}{5}$

Statement (s) is that both are aces.

$$P(A_2/5) = \frac{P(A_2).P(S/A_2)}{P(A_2).P(5/A_2) + P(\overline{A_2}).P(5/\overline{A_2})}$$

$$= \frac{\left(\frac{1}{221}\right) \times \left(\frac{3}{5}\right)}{\left(\frac{1}{221}\right) \times \left(\frac{3}{5}\right) + \left(\frac{220}{221}\right)\left(\frac{2}{5}\right)}$$

$$= \frac{3}{3 + (220 \times 2)}$$

$$= \frac{3}{443}$$

48. (C)

$$\int e^x \left(\frac{1 - \sin x}{1 - \cos x}\right) dx$$

$$= \int e^x \left(\frac{1 - 2\sin x/2 \cos x/2}{2\sin^2 x/2}\right) dx$$

$$\because \quad \frac{d}{dx}(-\cot x/2) = \frac{\cos ec^2 x/2}{2}$$

$$= e^x (-\cot x/2)$$

$$= -e^x \cot x/2$$

49. (B)

$$\begin{vmatrix} -bc & ca + ab & ca + ab \\ ab + bc & -ca & ab + \_bc \\ bc + ca & bc + ca & -ab \end{vmatrix}$$

$R_2 \to R_2 + R_1$ and $R_3 \to R_3 + R_1$

$$= \begin{vmatrix} -bc & ca + ab & ca + ab \\ ac & ab & 2ab + bc + ca \\ ca & ab + bc + 2ca & ca \end{vmatrix}$$

$C_3 \to C_3 - C_2$ and $C_2 \to C_2 - C_1$

$$= \begin{vmatrix} -bc & ab+bc+ca & 0 \\ ab & 0 & ab+bc+ca \\ ca & ab+bc+ca & -(ab+bc+ca) \end{vmatrix}$$

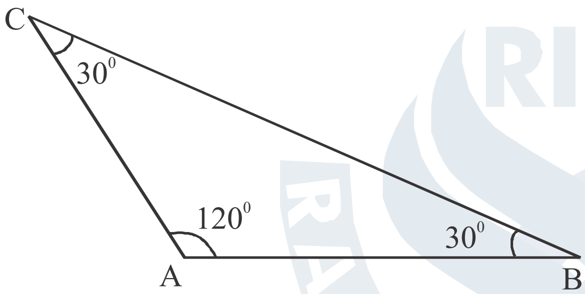
$$= (ab+bc+ca)^2 \begin{vmatrix} -bc & 1 & 0 \\ ab & 0 & 1 \\ ca & 1 & -1 \end{vmatrix}$$

$$= (ab+bc+ca)^2 \left[-bc(-1) - 1(-ab-ca)\right]$$

$$= (ab+bc+ca)^3$$

$$= \left(\sum ab\right)^3$$

50. (A)



$\because$ Angle in ratio 4 : 1 : 1
let, the nalge be 4x ; x and n

$\Rightarrow 6x = 180 \Rightarrow \boxed{x = 30}$

$$\frac{a}{\sin 120^0} = \frac{b}{\sin 30} = \frac{c}{\sin 30} = k$$

$$\Rightarrow a = \frac{\sqrt{3}\,k}{2}; b = \frac{k}{2}, c = \frac{k}{2}$$

$$\therefore \frac{a}{a+b+c} = \frac{\sqrt{3}/2}{\frac{\sqrt{3}}{2} + \frac{1}{2} + \frac{1}{2}} = \left(\frac{\sqrt{3}}{2+\sqrt{3}}\right)$$